p700  #1-8
p701  #1-13
In this investigation, you learned how to solve quadratic equations using the quadratic formula and to build a quadratic given its roots. These questions will help you summarize what you have learned.

1. Complete the following for each equation.
   - Make a table of inputs and outputs. Use at least five different inputs.
   - Sketch each graph on a separate grid using the table.
   - Find the points where each graph crosses the x-axis.
   a. \( x^2 - 2x - 4 = y \)  
   b. \( 3x^2 - 2x - 5 = y \)
   c. \( x^2 - 6x + 9 = y \)  
   d. \( 2x^2 + 4x + 3 = y \)

2. Find a value of \( k \) such that \( 2x^2 - 3x + k \) has each solution set.
   - a. two real-number solutions
   - b. one real-number solution
   - c. no real-number solutions

\[
X^2 - \frac{3}{2}X + \frac{k}{2} = \left( \frac{3}{4} \right)^2 - \frac{9}{16}
\]

\[
\frac{k}{2} = \frac{9}{16}
\]

\[
k = \frac{9}{8}
\]

3. Find a quadratic equation with the following roots.
   - a. 4 and \(-5\)
   - b. \(3 + \sqrt{2}\) and \(3 - \sqrt{2}\)

4. Factor the following nonmonic polynomials by writing them as monic polynomials in \( \mathbb{Z} \).
   - a. \(2x^2 - 5x - 12\)
   - b. \(6x^2 + 25x + 25\)
   - c. \(4x^2 + 8x - 5\)
   - d. \(9x^2 + 12x + 4\)

5. Factor the following quadratic expressions using the quadratic formula.
   - a. \(4x^2 + 12x + 5\)
   - b. \(6x^2 - 5x - 4\)

6. How can I solve any quadratic equation?

7. How can I factor any quadratic polynomial?

8. How are the roots of a quadratic equation related to its coefficients?

**Vocabulary**

In this investigation, you learned these terms. Make sure you understand what each one means and how to use it.

- algorithm
- monic
- nonmonic

Quadratic functions of the form \( f(x) = ax^2 + bx + c \) use various \( a, b, \) and \( c \) values to model different parabolic paths.
Mathematical Reflections  p. 700

1a–d. See graphs for input-output pairs.

1a. 
\[y = (x-1)^2 - 5\]

Crosses x-axis at \[x = 1 \pm \sqrt{5}\]

1b. 
\[y = x^2 - 4x - 5\]

Crosses x-axis at \[x = \frac{5}{3}\] and \[x = -1\]

1c. 
\[y = (x-3)^2 + 1\]

Meets x-axis at \[x = 3\].

1d. 
\[y = (x+3)^2 - 9\]

Does not cross x-axis.
Answers

Mathematical Reflections

1. See back of book.

2. a. \( k < \frac{9}{8} \)
   b. \( k = \frac{9}{8} \)
   c. \( k > \frac{9}{8} \)

3. a. \( x^2 + x - 20 = 0 \)
   b. \( x^2 - 6x + 7 = 0 \)

4. a. \( (2x + 3)(x - 4) \)
   b. \( (3x + 5)(2x + 5) \)
   c. \( (2x + 5)(2x - 1) \)
   d. \( (3x + 2)(3x + 2) \) or \( (3x + 2)^2 \)

5. a. \( (2x + 5)(2x + 1) \)
   b. \( (3x - 4)(2x + 1) \)

6. by using the quadratic formula

7. Answers may vary. Sample:
   Rewrite the polynomial as a monic polynomial in \( z \), and then factor that monic polynomial.

8. If the quadratic equation is \( ax^2 + bx + c = 0 \), then the sum of the roots is \( \frac{-b}{a} \) and the product is \( \frac{c}{a} \).
Multiple Choice

1. The equation \(5x^2 - 4x + k = 0\) has exactly two real-number solutions. Which of the following could NOT be the value of \(k\)?
   A. 0
   B. 1
   C. -1
   D. \(\frac{1}{2}\)

2. Which quadratic equation has roots \(1 + \sqrt{7}\) and \(1 - \sqrt{7}\)?
   A. \(x^2 + 7x - 1 = 0\)
   B. \(x^2 + 2x + 8 = 0\)
   C. \(x^2 - 2x - 6 = 0\)
   D. \(x^2 + 6x - 2 = 0\)

3. Which of the following are the solutions of \(3x^2 = x + 2\)?
   A. \(-\frac{2}{3}\) and 1
   B. 2 and 3
   C. 2 and -3
   D. \(\frac{2}{3}\) and -1

4. How many times does the graph of \((x - 2)^2 - 1 = y\) cross the x-axis?
   A. 0
   B. 1
   C. 2
   D. 3

5. Factor \(4x^2 + 5x - 6\).
   A. \((2x + 3)(2x - 2)\)
   B. \((4x + 3)(x - 2)\)
   C. \((2x + 1)(2x - 6)\)
   D. \((4x - 3)(x + 2)\)

6. Two positive integers have a sum of 25. Which of these could be their product?
   A. 10
   B. 26
   C. 46
   D. 25

Open Response

7. Use the quadratic formula to solve \(5x^2 - 3x - 1 = 0\).

8. a. Find two numbers with a sum that is 1 and a product that is -1.
   b. Verify that your results have a sum of 1 and a product of -1.

9. Find a quadratic equation with roots \(3 + 2\sqrt{2}\) and \(3 - 2\sqrt{2}\).

10. Factor \(6x^2 + 11x - 10\).

11. Solve \((x + 2)(2x - 7) = -15\).

12. Find the value of \(k\) such that \(9x^2 - 12x + k = 0\) will have exactly one solution.

Challenge Problem

13. The roots of the quadratic equation \(ax^2 + bx + c = 0\) are \(p\) and \(q\). The roots of the quadratic equation \(cx^2 + bx + a = 0\) are \(r\) and \(s\). \((a, b,\) and \(c\) are not zero). Compute \(pqr\), the product of all four roots.
\[ x = 3 \quad x = 5 \]

\[(x - 3)(x - 5)\]
\[ ax^2 + bx + c = 0 \]

Sum of Roots = \( -\frac{b}{a} \)

Product of Roots = \( \frac{c}{a} \)

Line Sym = \( \frac{-b}{2a} \)
Mid-Chapter Test

7. \( x = \frac{3 \pm \sqrt{29}}{10} \)
8. a. \( x = \frac{1 \pm \sqrt{5}}{2} \)
     b. \[ \frac{1 + \sqrt{5}}{2} + \frac{1 - \sqrt{5}}{2} = 1; \]
        \[ \left( \frac{1 + \sqrt{5}}{2} \right) \left( \frac{1 - \sqrt{5}}{2} \right) = -1 \]
9. \( x^2 - 6x + 1 = 0 \)
10. \( (3x - 2)(2x + 5) \)
11. \( x = 1 \) or \( x = \frac{1}{2} \)
12. \( k = 4 \)
13. \( pqrs = 1 \) as long as neither \( a \) nor \( c \) is zero.

\[ \frac{1}{2} + \frac{\sqrt{5}}{2} + \frac{1}{2} - \frac{\sqrt{5}}{2} = 1 \]
\[ \frac{1}{2} + \frac{1}{2} = 1 \]
\[ \frac{1 - \sqrt{5}}{4} = \frac{-1}{4} = -\frac{1}{4} \]