p755  #1-7
p758  #1-12
p760  #1-17
In Investigation 8A, you learned how to:

- use the quadratic formula to solve equations or determine whether an equation has no real solutions
- construct a quadratic equation given the equation’s two roots
- factor nonmonic quadratics

The following questions will help you check your understanding.

1. a. Solve the equation:
   \[84x^2 - 407x + 155 = 0.\]
   b. Factor over \(\mathbb{Z}\) the polynomial:
   \[84x^2 - 407x + 155.\]

2. Solve the following equations. If there are no real-number solutions, explain.
   a. \(x^2 - 5x - 14 = 0\)
   b. \(2x^2 + 5x + 3 = 0\)
   c. \(2x^2 + 5x = 4\)
   d. \(x^2 - x - 1 = x^2 - x + 1\)
   e. \((x + 3)(x - 4) = 8\)

3. Find a quadratic equation for each of the following pairs of roots.
   a. 3 and \(-1\)
   b. \(\frac{1}{2}\) and 5
   c. \(\sqrt{3}\) and \(-\sqrt{3}\)
   d. 0 and \(-5\)
   e. \(1 + \sqrt{2}\) and \(1 - \sqrt{2}\)
   f. \(\sqrt{2} + 1\) and \(\sqrt{2} - 1\)

4. Factor each quadratic polynomial twice. First, write the quadratic as a monic polynomial. Then use the quadratic formula.
   a. \(3x^2 + 11x + 10\)
   b. \(4x^2 + 4x - 15\)

\[3\left(\frac{x^2 + \frac{11}{3}x + \frac{10}{3}}{3}\right)\]
In **Investigation 8B**, you learned how to

- use your knowledge of quadratics to optimize some quadratic functions
- graph quadratic functions and examine the graph to find the vertex
- explore word problems involving quadratic functions

The following questions will help you check your understanding.

5. Each of the following describes a quadratic function. Find the vertex of each function.
   a. \( y + 3 = 2(x - 5)^2 \)
   b. \( y = (x + 1)(x - 3) \)
   c. \( y = 3x^2 + 18x + 8 \)
   d. \[
   \begin{array}{c|c}
   x & y \\
   \hline
   -2 & 9 \\
   0 & -15 \\
   1 & -21 \\
   2 & -23 \\
   4 & -15 \\
   6 & 9 \\
   \end{array}
   \]

6. There are many pairs of numbers that sum to 50.
   a. Which pair has the greatest product?
   b. What is that product?

7. A parabola has vertex \((4, -2)\) and includes the point \((3, -5)\).
   a. Use symmetry to identify one other point that must be on the graph of this function.
   b. Find an equation of the parabola.
   c. Sketch the graph of the parabola.
   d. How does this graph compare to the graph of \(y = x^2\)?
**Answers**

\[
\left( x - \frac{5}{12} \right) \left( x - \frac{21}{7} \right)
\]

\[
\left( 12x - 5 \right) \left( 7x - 3 \right)
\]

**Chapter Review**

1. a. \( x = \frac{5}{12} \), \( x = \frac{31}{7} \)
   
b. \((12x - 5)(7x - 31)\)
2. a. \( x = 7 \), \( x = -2 \)
   
b. \( x = -\frac{3}{2} \) or \( x = -1 \)
   
c. \( x = \frac{-5 \pm \sqrt{57}}{4} \)
   
d. no real solutions
   
e. \( x = 5 \) or \( x = -4 \)
4. a. \((3x + 5)(x + 2)\)
   
b. \((2x + 5)(2x - 3)\)
5. a. \((5, -3)\) \( b. (1, -4)\)
   
c. \((-3, -19)\) \( d. (2, -4)\)
6. a. 25 and 25 \( b. 625\)
7. a. \((5, -5)\)
   
b. \( y = -3(x - 4)^2 - 2 \)
   
c. Check student's graph.
   
d. Answers may vary. Sample: The graph is narrower than \( y = x^2 \), opens down, and is translated 4 units right and 2 units down.

**Review** pp. 758–759

3a. \( x^2 - 2x - 3 = 0 \)
3b. \( 2x^2 - 11x + 5 = 0 \)
3c. \( x^2 - 3 = 0 \) 3d. \( x^2 + 5x = 0 \)
3e. \( x^2 - 2x - 1 = 0 \)
3f. \( x^2 - 2\sqrt{x} - 1 = 0 \)
\[ x = \frac{5}{12} \quad x = \frac{31}{7} \]

\[
\left(x - \frac{5}{12}\right) \left(x - \frac{31}{7}\right) = 0
\]

\[
\left(12x - 5\right) \left(7x - 31\right) = 0
\]
8. Consider the quadratic function
   \[ y = -2x^2 + 4x - 3. \]
   a. Find the vertex and line of symmetry of this parabola.
   b. Find all three intercepts for the graph of this equation, if they exist.
   c. Sketch the graph of this equation.
   d. How does the graph of this equation compare to the graph of \( y = x^2 \)?

In Investigation 8C, you learned how to

- use the graphing method to solve or estimate the solutions of complex equations and inequalities
- sketch the solutions of inequalities of two variables and systems of inequalities of two variables
- use difference tables to analyze quadratics and other polynomials

The following questions will help you check your understanding.

9. Find a function that agrees with each table.
   a.
<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
</tr>
</tbody>
</table>

   b.
<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>1</td>
<td>-1\frac{1}{2}</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>-\frac{1}{2}</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>
10. Use the graphing method to approximate the solutions to each equation. Then solve the equation to find the exact solution(s).
   a. $2(x - 3) + 1 = -3x + 5$
   b. $3(x^2 - 1) = 2(x + 1)$

11. Draw graphs in the coordinate plane for the solutions of each of these inequalities or systems of inequalities.
   a. $y > 1$ and $x \leq 3$
   b. $y \geq -2x + 1$
   c. $x + y > 3$ and $2x - y < 4$
   d. $y > -x^2$
   e. $y \leq 2x^2$ and $y < x + 3$

12. Complete the difference table. Find a quadratic function that fits the table.

<table>
<thead>
<tr>
<th>Input, $x$</th>
<th>Input, $f(x)$</th>
<th>$\Delta$</th>
<th>$\Delta^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>31</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
8. a. (1, -1); \( x = 1 \)
   b. no x-intercepts; y-intercept is (0, -3)
   c. Check students' work.
   d. The graph is narrower than \( y = x^2 \), opens down, and is translated 1 unit right and 1 unit down.

9. a. \( y = 3x + 2 \)  
   b. \( y = \frac{1}{2}x - 2 \)

10. a. \( x = 2 \)
    b. \( x = \frac{5}{3}, x = -1 \)

\[ f(x) = 2x^2 - x + 3 \]


12. See Figure 42.
\[ f(x) = 2x^2 - x + 3 \]

### Figure 42

<table>
<thead>
<tr>
<th>Input ( x )</th>
<th>Output ( f(x) )</th>
<th>( \Delta )</th>
<th>( \Delta^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>31</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ 2A = 4 \]
\[ A = 2 \]

\[ 2x^2 + 17x + 0 \]

\[ 2x^2 - x + 3 \]
Multiple Choice

1. How many solutions does the quadratic equation $0 = 3x^2 - 7x - 13$ have?
   A. 0
   B. 1
   C. 2
   D. 3

2. In which quadrant is the vertex of the graph of $y = 3x^2 - 7x - 13$?
   A. I
   B. II
   C. III
   D. IV

3. Which of these points is NOT on the graph of the quadratic function $y = (x - 5)^2 + 10$?
   A. (0, −15)
   B. (5, 10)
   C. (10, 35)
   D. (15, 110)

4. For which value of $k$ does the quadratic equation $3x^2 - 24x + k = 0$ have exactly one real-number solution?
   A. −48
   B. −16
   C. 16
   D. 48

5. Which system of inequalities describes the shaded region below?
   A. $y \geq x^2 - 1$ and $y < 2x + 3$
   B. $y \leq x^2 - 1$ and $y > 2x + 3$
   C. $y \geq x^2 - 1$ and $y > 2x + 3$
   D. $y \leq x^2 - 1$ and $y < 2x + 3$

6. Two real numbers add up to 50. Which of the following numbers could be their product?
   A. −1000
   B. 1000
   C. 2000
   D. 3000

7. A quadratic function $f(x) = ax^2 + bx + c$ fits this table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
</tbody>
</table>

What is the value of $a$?
   A. $-1$  
   B. $\frac{1}{2}$
   C. 4  
   D. 8
### Answers

**Chapter Test**

1. C  
2. D  
3. A  
4. D  
5. A  
6. A  
7. C  
8. B  
9. B  
10. a. 75 ft  
     b. No; it more than doubles. Examples may vary.  
     c. about 68.1 miles per hour  
11. a. 5  

b. \( y = -3x^2 + 12x + 15 \)  
c. Check students’ work.
8. Which equation has solutions $2 + \sqrt{5}$ and $2 - \sqrt{5}$?
   A. $x^2 - 20 = 0$
   B. $x^2 - 4x - 1 = 0$
   C. $x^2 + 4x - 1 = 0$
   D. $x^2 - 4x - 21 = 0$

9. Which is the minimum value of the function $f(x) = x^2 - 12x + 33$?
   A. $-12$
   B. $-3$
   C. $3$
   D. $33$

10. Open Response

11. A parabola has vertex $(2, 27)$ and an $x$-intercept of $-1$.
    a. Use the symmetry of the parabola to find the other $x$-intercept.
    b. Find an equation for this parabola.
    c. Sketch the graph of the parabola.

12. Factor $6x^2 + 7x - 20$.

13. Find all the solutions to the equation $(2h + 3)(h - 4) = -13$.

14. Find a quadratic function to fit the table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-5</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>28</td>
</tr>
<tr>
<td>4</td>
<td>51</td>
</tr>
</tbody>
</table>

15. How are the roots of a quadratic equation related to its coefficients?

16. Consider the graph of the function $f(x) = x^2 - 8x + c$.
   a. If $c = 0$, find the vertex.
   b. Find a value of $c$ that makes the vertex $(4, 0)$.
   c. If $c = 24$, find the vertex.
   d. Find the coordinates of the vertex in terms of $c$.

17. Challenge Problem

   Use the equation below.
   $w(x) = (x + 3)(x + 1)(x - 2)(x - 4)$
   a. What is the degree of $w(x)$?
   b. Add columns to a difference table for $w(x)$ until you reach a column where the differences are constant.
   c. Predict how many difference columns you need to find a constant difference if the equation is quintic (degree 5).
12. \((2x + 5)(3x - 4)\)

13. \(h = \frac{5}{2}, h = 5\)

14. \(f(x) = 3x^2 + 2x - 5\)

15. Answers may vary, but should include the Sum and Product Theorem.

16. a. \((4, -16)\) \hspace{1cm} b. \(16\) \hspace{1cm} c. \((4, 8)\) \hspace{1cm} d. \((4, c - 16)\)

17. a. degree 4
b. See back of book.
c. You will need a \(\Delta^5\) column in which the differences would be constant.

<table>
<thead>
<tr>
<th>Test</th>
<th>pp. 760–761</th>
</tr>
</thead>
<tbody>
<tr>
<td>17b.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x))</th>
<th>(\Delta)</th>
<th>(\Delta^2)</th>
<th>(\Delta^3)</th>
<th>(\Delta^4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-4)</td>
<td>144</td>
<td>-144</td>
<td>120</td>
<td>-72</td>
<td>24</td>
</tr>
<tr>
<td>(-3)</td>
<td>0</td>
<td>-24</td>
<td>48</td>
<td>-48</td>
<td>24</td>
</tr>
<tr>
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<td>0</td>
<td>-24</td>
<td>24</td>
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<td>24</td>
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