You can use quadratic functions to model real-world situations. Optimization problems use quadratic functions to determine the greatest area for a rectangular region.

For You to Explore

1. You have 200 feet of fencing to use for building a rectangular dog pen. Find the dimensions of the pen having the greatest possible area.

2. Suppose you build the pen such that it borders an apartment building. You will only need to build three walls. Find the dimensions of the pen having the greatest possible area.

3. Suppose that you have three pets, a dog, a cat, and a monkey. You want to divide the pen into three smaller rectangular sections. There will be one pen for each animal. The pens will be side by side as shown below.

You still only have 200 feet of fencing. You will have to use some of the fencing to build the dividers between the pens. You want to maximize the area of the entire pen. What should the dimensions be?
You can use quadratic functions to model real-world situations. Optimization problems use quadratic functions to determine the greatest area for a rectangular region.

For You to Explore

1. You have 200 feet of fencing to use for building a rectangular dog pen. Find the dimensions of the pen having the greatest possible area.

   \[ A = L \cdot W \]
   \[ L = W \]
   \[ A = W^2 \]
   \[ P = 2L + 2W \]
   \[ = 2W + 2W \]
   \[ 200 = 4W \]
   \[ 50 = W \]

2. Suppose you build the pen such that it borders an apartment building. You will only need to build three walls. Find the dimensions of the pen having the greatest possible area.

3. Suppose that you have three pets, a dog, a cat, and a monkey. You want to divide the pen into three smaller rectangular sections. There will be one pen for each animal. The pens will be side by side as shown below.

You still only have 200 feet of fencing. You will have to use some of the fencing to build the dividers between the pens. You want to maximize the area of the entire pen. What should the dimensions be?
Answers

For You to Explore

1. 50 ft by 50 ft
2. 50 ft by 100 ft
3. 25 ft by 50 ft
4. Suppose you build a pen to separate five monkeys. You have 600 feet of fencing. What dimensions give the maximum area for the entire pen?

5. **Take It Further** Explain the pattern to the solutions of Exercises 1–4 in as much detail as possible.

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**Answers**

4. 50 ft by 150 ft

5. The total amount of fence used for the vertical sides is the same as the total used for the horizontal sides.
4. Suppose you build a pen to separate five monkeys. You have 600 feet of fencing. What dimensions give the maximum area for the entire pen?

\[2y + 6y = 600\]
\[x = 3y\]

5. **Take It Further** Explain the pattern to the solutions of Exercises 1–4 in as much detail as possible.

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**Answers**

4. 50 ft by 150 ft

5. The total amount of fence used for the vertical sides is the same as the total used for the horizontal sides.
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On Your Own

6. What is the least possible value of the function $f(x) = x^2 - 6x + 8$?

7. Suppose $f(x) = 3 - (x - 2)^2$. Find the value of each expression.
   a. $f(5) - f(-1)$
   b. $f(8) - f(-4)$
   c. $f(3) - f(1)$
   d. $f(2 + \sqrt{17}) - f(2 - \sqrt{17})$

8. Solve these equations using any method. Check each result.
   a. $x^2 + 14x + 40 = 0$
   b. $x^2 - 5x = 24$
   c. $9x^2 = 49$
   d. $x^3 - 4x = 0$

9. Does the graph of $y = (3x + 5)^2 + \frac{1}{3}$ intersect the $x$-axis? If so, where is the intersection? If not, explain.
8. Solve these equations using any method. Check each result.

a. \( x^2 + 14x + 40 = 0 \)

b. \( x^2 - 5x = 24 \)

c. \( 9x^2 = 49 \)

d. \( x^3 - 4x = 0 \)

\[
\begin{align*}
x^2 - 5x - 74 &= 0 \\
5 \pm \sqrt{25 - (4 \cdot 24)} &= 0 \\
5 \pm \sqrt{121} &= 0 \\
5 \pm 11 &= 0 \\
\frac{5 + 11}{2} &= \frac{16}{2} \quad \text{or} \quad \frac{-5}{2} \\
= 8 \quad \text{or} -3 \\
\end{align*}
\]
Exercises

6. $-1$

7. **a–d.** All values are 0.

8. a. $x = -10$ or $x = -4$
   
   b. $x = 8$ or $x = -3$

   c. $x = \pm \frac{7}{3}$

   d. $x = 0$, 2, or $-2$

9. The graph does not intersect the $x$-axis. Explanations may vary.
Maintain Your Skills

For Exercises 10–12, find the least possible value of each function. Find the value of $x$ for which the least value occurs.

10. a. $a(x) = x^2$
   b. $b(x) = (x - 1)^2$
   c. $c(x) = (x + 5)^2$
   d. $d(x) = (x - \sqrt{13})^2$
   e. $e(x) = (x - k)^2$, where $k$ is a constant

11. a. $f(x) = x^2$
   b. $g(x) = x^2 + 5$
   c. $h(x) = x^2 - 17$
   d. $j(x) = x^2 - \sqrt{13}$
   e. $k(x) = x^2 + r$, where $r$ is a constant

12. a. $l(x) = (x - 5)^2$
   b. $m(x) = (x - 3)^2 + 2$
   c. $n(x) = (x + 17)^2 - 11$
   d. $p(x) = \left(x - \frac{5}{3}\right)^2 - \sqrt{13}$
   e. $q(x) = (x - h)^2 + k$, where $h$ and $k$ are constants
10. a–e. The minimum is at the given point.
   a. (0, 0)
   b. (1, 0)
   c. (−5, 0)
   d. (\sqrt{13}, 0)
   e. (k, 0)

11. a–e. The minimum is at the given point.
   a. (0, 0)
   b. (0, 5)
   c. (0, −17)
   d. (0, −\sqrt{13})
   e. (0, r)

12. a–e. The minimum is at the given point.
   a. (5, 0)
   b. (3, 2)
   c. (−17, −11)
   d. \left( \frac{5}{3}, −\sqrt{13} \right)
   e. (h, k)
12. a–e. The minimum is at the given point.
   a. (5, 0)           d. \( \left( \frac{5}{3}, -\sqrt{13} \right) \)
   b. (3, 2)           e. (h, k)
   c. (−17, −11)