Read p 739-744
p744 #1-5
In Lesson 4.14, you looked at strategies for solving inequalities. Those strategies incorporated Tony’s method from Lesson 8.10. They also provided a way to solve inequalities of all sorts, even daunting inequalities such as $2x^2 - 21x + 24 > -7\sqrt{2x + 3}$.

Solving such an inequality can be very tricky. You can use the graphing method. You may not find the exact solution, but you will get a good estimate for the solution.
$x > 3$

\[
\begin{array}{c}
\text{0} \\
\text{3}
\end{array}
\]

\[
\begin{array}{c}
\text{0} \\
\text{3}
\end{array}
\]
$y > 2x$

$y > x^2$
For You to Do

1. Use the graphing method to solve the inequality
   \[2x^2 - 21x + 24 > -7\sqrt{2x} + 3.\]
All of the inequalities you have studied so far have had only one variable. You can visualize the solution set of one-variable inequalities by graphing the solution on a number line.

\[ x < 3 \]
\[ x > 5 \]

To visualize the solution set of a two-variable inequality, you can graph it on the coordinate plane.
Example 1

Problem  Graph the solution set to the inequality \( y < 3x + 5 \).

Solution  First, you graph the corresponding equation, \( y = 3x + 5 \). Notice that the inequality is \(<\), and not \(\leq\). So points along the line \( y = 3x + 5 \) will not be part of the solution set. You can indicate that the line is not in your solution set by drawing a dashed line.

Think of each point that is on the line as having coordinates of the form \((x, 3x + 5)\). You want all the points that satisfy \( y < 3x + 5 \). So for any \( x \), you want the value of the \( y \)-coordinate to be less than \( 3x + 5 \).

Try \( x = 0 \). The point on the line with \( x \)-coordinate 0 would be \((0, 5)\). Any point on the vertical line \( x = 0 \) with \( y \)-coordinate less than 5 is part of the solution set.

Next, try \( x = 5 \). The point on the line with \( x \)-coordinate 5 is \((5, 20)\), and any point with \( x \)-coordinate 5 and \( y \)-coordinate less than 20 is part of the solution set.

It would take you forever to write out the situation for every possible value of \( x \), but you can see that any point that is below the line is part of the solution set.
Just as you had systems of equations in Chapter 4, you can also have systems of inequalities. Tony’s graphing method helps with these problems, too.
Minds in Action  episode 44

Tony and Sasha are having a conversation about a movie they showed to raise money for their school’s highly successful math team.

Tony  What a turnout for the movie. Over 100 people showed up.
Sasha  Yes, that was great! We still didn’t make it to our $500 goal, though.
Tony  Really? I can’t believe that’s possible. There were more than 100 people! We charged $8 per adult and $4 per student, and there were some of each.
Sasha  Here, make a graph of it. Let’s say $a$ is the number of adult tickets, and $s$ is the number of student tickets. We want to see what the graph of $8a + 4s < 500$ looks like. Let’s start by graphing the line $8a + 4s = 500$. 
Tony pulls out some graph paper and quickly sketches this graph.

\[ 8a + 4s = 500 \]

**Sasha** Since we didn’t make $500, we must be below that line.

**Tony** How do you know that?

**Sasha** Think about it. If we made $480, the line would be \( 8a + 4s = 480 \). If we made $200, the line would be \( 8a + 4s = 200 \). Here, I’ll draw those lines.

So \( 8a + 4s = \) anything less than $500 would be below that line.
Tony  Okay, I get it. But I still don’t think we can be below that first line with over 100 people paying.

Sasha  Look, we had more than 100 people, so let’s add \( a + s > 100 \) to the graph. Here’s the line \( a + s = 100 \).

\[ \begin{align*}
8a + 4s &= 500 \\
\text{Number of Adult Tickets} \\
\text{Number of Student Tickets} \\
\end{align*} \]

Since we had more than 100 people, we had to be above this line. And we know we’re below the graph of \( 8a + 4s = 500 \).

Sasha shades in the graph.

Sasha  And see that little corner? Our sales must be somewhere in that shaded part. That’s what happens when you let students in for half price.

Tony  Maybe next year we should charge all of them five dollars.
For Discussion

2. Using algebra, find the intersection of the two lines that Sasha graphed.
3. Are there any other restrictions on the values of $a$ and $s$?

\[
8A + 4s = 500 \\
8A + 4s = 100
\]

\[
-4A - 4s = -400
\]

\[
A = 25 \\
A + s = 100 \\
25 + s = 100 \\
s = 75
\]

Answers

For Discussion

2. $a = 25$, $s = 75$
3. Answers may vary. Sample: They have to be whole numbers because they represent a number of people.
As with inequalities of one variable, the solution set for Problem 2 includes more than one number. It is a portion (or portions) of the coordinate plane, instead of a portion of the number line. Graphs describe sets. A shaded region indicates that any point in that region satisfies the inequality. The boundaries of solution regions do not have to be straight lines.
Example 2

**Problem** Draw a graph with all points that are solutions to both inequalities.

\[ y \geq x^2 \quad \text{and} \quad y < 6 - x \]

**Solution** Start by sketching the graphs of 

\[ y = x^2 \quad \text{and} \quad y = 6 - x \]

The y-coordinate of any point *above* the graph of \( y \geq x^2 \) is greater than the y-coordinate of the point *on* the graph with the same x-coordinate. So all the points above the graph are part of the solution set. Similarly, for \( y < 6 - x \), any point *below* the line is part of the solution set.

To satisfy both inequalities, a point has to be above the graph of \( y = x^2 \) and below the graph of \( y = 6 - x \).
For Discussion

4. What does the solution for this system of inequalities look like?

\[ y \geq x^2 \quad \text{and} \quad y < -6 - x \]
For You to Do

5. Using your graphing calculator, sketch the solution set for this system of inequalities.

\[ y \geq 2x^2 - 21x + 24 \quad \text{and} \quad y \leq -7\sqrt{2x + 3} \]

For the shaded region, \( 3 \leq x \leq 6.5 \).
Check Your Understanding

1. For the next showing of a movie, Tony decides to charge $5 per student. Adults will still pay $8. His goal is to raise at least $500. The inequality $8a + 5s \geq 500$ describes this goal.
   a. Graph the line with equation $8a + 5s = 500$.
   b. Graph the solution to the inequality $8a + 5s \geq 500$.

2. Tony expects that more than 100 people will attend the next showing. Graph the solution to the system of inequalities.

   $a + s > 100$ and $8a + 5s \geq 500$

   Can Tony be sure the movie will make $500 if more than 100 people attend?

3. Draw graphs in the coordinate plane for the solutions of each of these inequalities or systems of inequalities.
   a. $y \leq 5$
   b. $y \geq 0$ and $x \geq 0$
   c. $y > -3$ and $x \geq 4$
   d. $y > 2x - 5$
   e. $2x + 3y < 12$ and $2x + y < 8$
   f. $y \geq x^2$
   g. $y \geq x^2$ and $y \leq 4$
   h. $y \geq |x|$ and $y < -3$
2. See Figure 40.

yes
h. \( y \geq |x| \) and \( y < -3 \)
4. **Take It Further** Graphing the inequality \( \frac{y - 4}{x - 3} \geq 2 \) is difficult, since \( x \) cannot be 3.

a. Why can't \( x \) equal 3 in this inequality?

b. Draw \( \frac{y - 4}{x - 3} = 2 \) and \( x = 3 \) on a coordinate plane. Decide whether you should draw each equation as a solid or dashed line.

c. Choose convenient points in all four zones to draw the solution set for the inequality.

5. Derman looks at Exercise 4 and says, "Hmm, this is the equation for slope. So it's telling me I want a line through (3, 4) that has a slope of at least 2. There are many lines like that."

a. Draw a few lines that go through the point (3, 4) and have a slope of at least 2. A slope of at least 2 means that there are no negative slopes!

b. How steep can these lines be? Is there a boundary?

c. **Take It Further** What does the set of these lines look like?
p745  #6-12
6. a. On the same coordinate plane, graph the two lines $y = x + 1$ and $y = -2x + 11$.
   
   b. Find the value of $x$ such that $x + 1 = -2x + 11$. What does this value of $x$ represent?
   
   c. Find two points $(x, y)$ such that $y > x + 1$ and $y < -2x + 11$. Where are these points located on the graph?
   
   d. Find two points $(x, y)$ such that $y > x + 1$ and $y > -2x + 11$. Where are these points located on the graph?
   
   e. Shade the entire region where $y < x + 1$ and $y > -2x + 11$ are both true. (Hint: You might start by finding some points that make both these inequalities true.)

![Graph of two lines and shaded region]
Exercises 7 and 8 refer to Exercise 3.

7. In Exercise 3(e), you graphed an inequality involving the following lines.

\[ 2x + 3y = 12 \]
\[ \underline{2x + y = 8} \]

Use algebra to find the intersection of the two lines. Then check your result by testing the values of \( x \) and \( y \) in each equation.

\[ 2x + 2 = 8 \]
\[ 2x = 6 \]
\[ x = 3 \]

7. \((3, 2)\)
8. a. Sketch the solution set of these inequalities.

\[2(x - 3) + 3y < 12 \quad \text{and} \quad 2(x - 3) + y < 8\]

b. What is the intersection point of the corresponding equations?

c. How is the graph in part (a) related to the graph in Exercise 3(e)?

d. Predict the intersection point of the lines \(2(x - 8) + 3y = 12\) and \(2(x - 8) + y = 8\). Then check your prediction.

e. Predict the intersection point of the lines \(2(x + 8) + 3y = 12\) and \(2(x + 8) + y = 8\). Then check your prediction.

f. Sketch the solution set of these inequalities.

\[2x + 3(y - 4) < 12 \quad \text{and} \quad 2x + (y - 4) < 8\]

Find the new intersection point.

g. Predict the intersection point of the lines \(2(x - 4) + 3(y + 5) = 12\) and \(2(x - 4) + (y + 5) = 8\). Then check your prediction.

**Answers**

8. a. [Diagram showing the solution set for the inequalities given in part (a).]

b. \((6, 2)\)

c. The entire graph is 3 units to the right of the previous graph.

d. \((11, 2)\)

e. \((-5, 2)\)

f. [Diagram showing the new intersection point for the inequalities given in part (f).]

g. \((7, -3)\)
\[
2(x-3) + 3y < 12
\]
\[
3y < -2(x-3) + 12
\]
\[
y < -\frac{2(x-3)}{3} + \frac{12}{3}
\]
\[
y < -\frac{2x+6}{3} + 4
\]
\[
y < -\frac{2}{3}x + 2 + 4
\]
\[
y < -\frac{2}{3}x + 6
\]
9. **Standardized Test Prep** Which point is NOT in the intersection of \( y > x^2 + 5x + 6 \) and \( y \leq 4 \)?

A. \((-4, 3)\)  
B. \((-3, 4)\)  
C. \((-2, 0)\)  
D. \((-1, 3)\)
10. a. Graph \( y > |x| \). Where is the shaded area relative to the graph of \( y = |x| \)?

b. Use the graph of \( y = x^3 - x \) to draw a graph of \( y \geq x^3 - x \).

c. If an inequality is in the form \( y > f(x) \), do you shade above or below the graph of the equation \( y = f(x) \)? Explain.

The shaded region is above the graph.

c. above; the y-value of a solution is greater than the corresponding \( f(x) \) value.
11. **Take It Further** Sketch each graph. You might start by finding some points that make the inequality work, or by finding the boundaries.

a. \( |x| + |y| \leq 2 \)

\[ |x| + |y| = 2 \]

b. \( |x + y| \leq 2 \)

\[ x + y \leq 2 \]
\[ x + y \geq -2 \]

c. \( |x| - |y| \leq 1 \)

d. \( \frac{|y|}{|x|} \geq 1 \)
Maintain Your Skills

12. Graph the solution set for each inequality.
   a. \( y - 5 > 3(x + 2) \)
   b. \( y - 5 \geq 2(x + 2) \)
   c. \( y - 5 < \frac{1}{2}(x + 2) \)
   d. \( y - 5 \leq -\frac{1}{3}(x + 2) \)