



Parent FAQ
Frequently Asked Questions About K-5 Math

1. **What are the characteristics of the district's math program?**
2. **What if my child is struggling in math class?**
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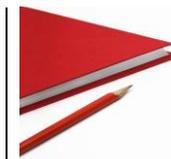
1. What are the characteristics of the district's math program?

Traits of the *Math in Focus* approach include:

- Building understanding of concepts with a Concrete-Pictorial-Abstract learning progression (C-P-A)
- Encouraging all learners with built-in differentiation support.
 - Math in Focus materials and lesson design include resources and adaptations for on-level, advanced and struggling students as well as English Learners.
 - Materials also include a range of assessments, from formative (informal checks for understanding/mastery) to summative (formal mastery) to assist in identifying appropriate levels of challenge.
- Bringing the instructional practices foundational to Singapore Math's high achievement to U.S. students (Singapore consistently ranks at the top in math achievement on international measures (TIMMS,1994-2015)). These practices include:
 - Visualization: Concrete-pictorial-abstract learning progression, use of consistent visual models to support foundational and extended learning.
 - Gradual release model: student-focused instruction, guided learning, independent practice with release to tackle routine and non-routine problems and deepen understanding.
 - Scaffolded content: Instruction is adapted for needs of individual learners (with differentiated sequence and prompts).

Program outcomes are described as follows:

- To build conceptual understanding (through Concrete-Pictorial-Abstract (CPA) approaches)
- To develop critical thinking skills (by applying math concepts in different ways, creating alternative solutions to problems)



- To advance problem-solving proficiency (applying different strategies, considering best strategies for routine and non-routine problems)
- To build positive attitudes (learning to approach problem-solving with interest, enthusiasm, and confidence to persevere).

(Houghton Mifflin Harcourt, 2018)

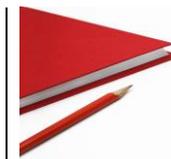
At the time of the district selection of the math program, the standards for math instruction were changing to the new PA Core Standards. The Math Vertical Team considered the increasing demands of the new standards and recognized the profound shift in focus from content coverage and algorithmic/rote mastery to what the standards refer to as “*Habits of Mind of a Productive Mathematical Thinker*” (PDE, 2014). These practices relate to student reasoning and communication of reasoning, modeling and using tools, seeing structure and generalizing. Successful mastery of grade level content standards requires synchronous development of math habits of mind. One of the greatest shifts with the new standards and the core program is the requirement to explain reasoning and engage in mathematical discourse to analyze approaches, relationships and ideas. Student talk about math has become a new and highly important element of math instruction and student learning and enrichment. Algorithmic mastery is not sufficient as the sole or primary indicator of mathematical understanding.

The standards for mathematical practices are listed below:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
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4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

By middle school and high school, the state standards “set a rigorous definition of college and career readiness by demanding that students develop a depth of understanding and ability to apply mathematics to novel situations, as college students and employees regularly do” (PDE, 2014). Secondary grade level student success in higher level mathematics will depend on a strong elementary foundation with deep conceptual understanding as well as procedural knowledge. The *Math in Focus* program includes a progression of problem-solving, novel problems and application that match the increased demands of the new standards.

In addition to aligning to the rigorous instructional and learning practices of the new standards, *Math in Focus* aligns to the standards and principles of the National Council of Teachers of Mathematics (NCTM), which reinforce the idea of developing depth of understanding along with procedural fluency (HMH, 2018; NCTM, 2018). During the 2011-2012 program review, the Math Vertical Team developed a program mission and identified specific areas of student focus to support the development of strong habits that develop mathematical practices. The program mission and student focus areas are listed below.



WSSD Mathematics Program Mission

The mission of the Wallingford/Swarthmore School District mathematics program is to:

1. Inspire intellectual curiosity and aesthetic appreciation of mathematics as a body of knowledge
2. Promote critical thinking and problem solving skills to solve real world problems
3. Develop computational fluency and understanding of mathematical algorithms in order to apply known facts to unknown situations
4. Encourage students to be active participants in a life-long learning process as independent thinkers

WSSD Mathematics Student Focus

In order to accomplish our mathematics mission, the WSSD mathematics program should encourage students to:

1. Understand that mathematical thinking is as much about the questions as it is about creatively uncovering patterns that lead to answers
2. Take ownership of learning and develop skills needed for self-evaluation
3. Discover concepts through experience before executing algorithms
4. Use multiple approaches to problem solving
5. Understand developmentally appropriate mathematical ideas and apply this knowledge to solve problems
6. Enjoy mathematics while celebrating the disequilibrium that is a natural part of the learning process
7. Work collaboratively and cooperatively to solve problems, using mathematical language to explain thought processes

2. What if my child is struggling in math class?

Wallingford-Swarthmore School District uses the *Math in Focus* program, which, when implemented with fidelity, is a research-based program for students across the learning continuum. *Math in Focus* is a rich comprehensive math program that supports development of mathematical problem-solving. *Math in Focus* is the United States edition of Singapore Math, a highly effective curriculum that integrates best practices in math instruction and centers on problem-solving as the focus of mathematical learning. The approach and program resources are based on the more than twenty years of success experienced by Singapore as a world leader in math. There is positive district-wide feedback about the level of challenge and engagement our students experience in math.

Traits of the *Math in Focus* approach include:

- Building understanding of concepts with a Concrete-Pictorial-Abstract learning progression (C-P-A)
- Encouraging all learners with built-in differentiation support.
 - Math in Focus materials and lesson design include resources and adaptations for on-level, advanced and struggling students as well as English Learners.
 - Materials also include a range of assessments, from formative (informal checks for understanding/mastery) to summative (formal mastery) to assist in identifying appropriate levels of challenge.



- Bringing the instructional practices foundational to Singapore Math’s high achievement to U.S. students (Singapore consistently ranks at the top in math achievement on international measures (TIMMS,1994-2015)). These practices include:
 - Visualization: Concrete-pictorial-abstract learning progression, use of consistent visual models to support foundational and extended learning.
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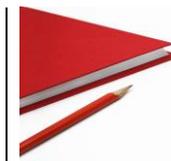
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(Houghton Mifflin Harcourt, 2018)

Consider whether your child is struggling or actually just still growing in certain areas. Percentage-based scores are not always the best way to interpret mastery in a standards-based program. Children develop at different rates and can demonstrate proficiency at a competency level or an advanced level on different skills throughout the year. Some students demonstrate proficiency in meeting the rigorous grade level standard and they show this with successful mastery of targeted assessment items. Assessments (tests) include novel items, which present tasks in ways that students have never seen, or that ask students to apply learning in ways they have never tried before. When students show difficulty on those items, they show they are still growing in that area, but they may have still shown mastery of the standard by successful completion of other challenging standards-aligned items on the test.

When teachers communicate student progress at conference time and on district progress reports, progress is reported in relation to student mastery of the standards. Student progress is described as *Meets Expectations*, *Exceeds Expectations*, or *Approaching Expectations*. Traditional percentage-based scores equating with the traditional A through F scale do not apply to a growth model which is critical in elementary school. If concerns have emerged because of a percentage-based interpretation of mastery that aligns to the traditional A through F scale, it might not be an accurate interpretation of mastery. A more important question would be “Can my child succeed on the items that show mastery of the standard?”

If a teacher has communicated concern about student mastery of the grade level standards, we have systems of support. In addition to rich and varied *Math in Focus* resources used in class and beyond to support struggling learners, our district uses an intervention block to provide additional targeted support for students who need additional time to master foundation skills. During the intervention block, students might have the opportunity for extension and support for current units using *Math in Focus* resources for differentiation. Students might also or instead have the opportunity for practice and instruction using supplemental *Do the Math* resources. *Do the Math* is a carefully sequenced program designed to provide targeted



instruction in important foundation skills and conceptual understanding in number sense, addition, subtraction, multiplication and division. The district uses common data team practices across all buildings to evaluate student needs and tailor instruction to support development of student mastery and personal excellence. Parents are important partners in supporting student learning. If you have concerns about your child's progress, make sure to communicate with your child's teacher or your school's principal.

3. What if my child has advanced skills in math?

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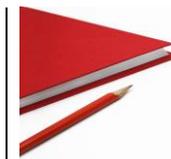
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In addition to rich and varied resources for extension, problem-solving and enrichment, our teachers supplement instruction with additional challenge resources to further enrich the



mathematical development of students. The schoolwide enrichment model supports increased access to enriched experiences for all students whose mastery and readiness suggest a need for additional individualization. Parents are important partners in supporting student learning. If you have concerns about your child's appropriate level of challenge, make sure to communicate with your child's teacher or principal.

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4. How do I know if my child should be accelerated?

Students demonstrating advanced mastery may benefit from acceleration. It is important to consider what mastery means and the many types of acceleration there can be.

Mastery

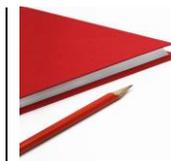
Mastery encompasses more than rote or procedural knowledge. It includes depth of conceptual understanding as well as development of mathematical practices. The increasing demands of the new standards have led to profound shift in focus from content coverage and algorithmic/rote mastery to what the standards refer to as "*Habits of Mind of a Productive Mathematical Thinker*" (PDE, 2014). These practices relate to student reasoning and communication of reasoning, modeling and using tools, seeing structure and generalizing. Successful mastery of grade level content standards requires synchronous development of math habits of mind. One of the greatest shifts with the new standards and the core program is the requirement to explain reasoning and engage in mathematical discourse to analyze approaches, relationships and ideas. Student talk about math has become a new and highly important element of math instruction and student learning and enrichment. Algorithmic mastery is not sufficient as the sole or primary indicator of mathematical understanding.

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understanding along with procedural fluency (HMH, 2018; NCTM, 2018). During the 2011-2012 program review, the Math Vertical Team developed a program mission and identified specific areas of student focus to support the development of strong habits that develop mathematical practices.

To have a deeper understanding of the type of problem-solving that will the support advanced problem-solving required in STEM and engineering careers of the future, consider viewing the engaging Ted Talks by math teacher Dan Meyer, or explore some of the additional resources for further reading provided on the Curriculum and Education website.

Acceleration

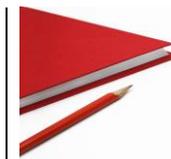
For many people, “acceleration” is synonymous with "grade skipping," although grade skipping is just one type of acceleration. There are at least twenty types according to some resources. We would not suggest acceleration is just moving faster through material. We want to ensure a strong foundation with depth of understanding of conceptual knowledge as well as procedural fluency. The real world problem solving of the future will require stamina, flexible mathematical thinking and strong sense of mathematical reasoning, argumentation and communication. Our current program has a high ceiling. It is very complex. When we do have students whose mastery suggests some form of acceleration is appropriate, the strategies most frequently used include continuous progress (differentiation based on mastery), content acceleration, curriculum compacting, or mentoring. When appropriate for certain students, we do employ grade level acceleration. With adolescent brain development and growth in abstract thinking, some of our older advanced students really do have readiness for even greater acceleration than is offered in the regular math classroom through individual or small group differentiation. We have more widespread ways of offering that for our more mature students when they reach middle school.

What is important about acceleration is identifying the appropriate match of enrichment strategy based on student classroom performance. The district maintains a commitment across all levels to continuous improvement in our ability to identify student enrichment needs, and a real dedication to meeting the needs of advanced students (GIEP or non-GIEP).

For additional information, see the Davidson Institute, or the Acceleration Institute for definitions of the many possible types of acceleration that may be appropriate for different students based on individual needs.

5. What if my child reports being bored in math?

Parents are important partners in supporting student learning. If a child is going home reporting boredom, it is important to communicate with your child’s teacher and work together to find out if boredom is really the issue. As many parents know, there can be many reasons a child may report being bored. Occasionally, children may report boredom if tasks are not personally appealing, no matter the subject. Some children may also approach a task with minimal effort or engagement. This can impact the appeal of the task and the feeling of engagement. Additionally, children who fear risk, or find a task uncomfortably challenging, can report boredom as a way to save face rather than admit difficulty. Sometimes adaptations that increase choice, interest, or support can help a student more fully engage in a non-preferred task. Parent-teacher communication can help get to the root of the issue.



Children who have a rapid rate of retention may not need as much repetition or practice on some things. One of the challenges of advanced math mastery is that students need to develop the ability to explain mathematical reasoning and develop mathematical arguments. Sometimes students who are exclusively focused on procedurally getting a right answer may find it less interesting to explain or defend reasoning or find alternate approaches. That can sometimes indicate the discomfort of disequilibrium, or the discomfort of cognitive challenge that means learning is taking place. Advanced students might not like that feeling and report the task as boring, when it could be an indication of challenge.

If a child has depth of understanding and mastery already and reports boredom, the teacher will likely have data that shows a higher level of mastery and appropriate adaptations can be made. Additional assessments may take place as necessary to determine the right fit for an advanced student. Enrichment coordinators can also work with teachers to identify enrichment needs.

6. Will my child be prepared for advanced opportunities in middle or high school?

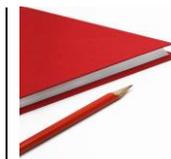
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The elementary program provides a strong foundation for students to prepare them for the increasing complexity of middle school and high school math. We also use a vertical (cross grade) team model to ensure alignment of student experience with teacher expectations across the grades.



7. What if my child is not ready for the enriched class in 5th grade but wants to participate in enriched or advanced classes in middle or high school?

Children develop at different rates and our math program provides opportunities for differentiation and enrichment throughout the grades. Adolescent development of abstract thinking can be as variable as early childhood development. As students develop as mathematical thinkers and problem-solvers, there are many opportunities to increase levels of challenge.

If students do not show readiness at the end of fourth grade for placement in an enriched 5th grade class, the fifth grade level team continuously monitors student progress and can differentiate for students to increase access to enrichment or change a placement if it seems appropriate for a student. The same process is followed at the middle school. Many students move up from an on-level class to an enriched or accelerated experience as they move through middle school.

It is also important to remember that with the block scheduling at the high school, students with or without an early enriched placement can still take advanced terminal levels of math by the end of high school (Calculus or beyond). The high school transcript will not show elementary or middle school math levels, only the terminal math courses and achievement in high school. The strong elementary academic, intellectual, social and emotional foundation will help ensure the readiness for increasing challenge and complexity with a strong sense of self-efficacy for the future.

8. How does the district monitor the success of a new practice, such as a change in grouping practices?

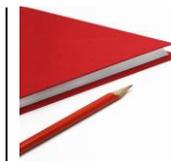
Grade level and building teams engage in ongoing review of existing practices, best practices and student performance data, all of which are a continuation of the curriculum monitoring process that led to the current review of practices and culminated in recommendations for improvement through revision to the relatively new practice of elementary ability grouping. Representative leadership teams, that include teachers and administrators, work collaboratively to monitor and support. This is an ongoing process that is part of the continuous improvement model of our Curriculum Review Cycle.

9. What is the district process for curricular or instructional decisions?

The vertical team process as part of the Curriculum Review Cycle ensures ongoing representative teacher input in decisions. The representative nature of the process allows teacher representatives to collect teacher concerns for discussion and input and to be addressed through the final recommendations. Part of the Curriculum Review Cycle across all subject areas involves maintaining feedback loops between teachers and vertical team representatives. Final recommendations are made by the vertical leadership team that has developed greater best practice professional knowledge through the vertical team process itself, as they evaluate existing practice and check against best practice research and in consultation with math and program specialists.

10. What is the role of parent input in instructional decisions such as ability grouping?

Parents are critical partners in supporting the learning of students. Typically instructional design, placement and scheduling are administrative decisions guided by a representative teacher and administrative curriculum leadership team. In the case of ability grouping, a team took a year to study and discuss best practice after multi-year monitoring and data review. Overall, it was a rigorous multi-year process engaged in by those with professional expertise,



expertise that was increased through the process itself, and checked against best practice research and in consultation with math and program specialists.

Parent communication and education will be part of the transition phases. The curriculum update to the board provides information through the public board meeting. There will be a follow-up K-5 parent information night on May 30th, to revisit the traits and approach of the program and its design for supporting both advanced and struggling students. Additionally, there is a plan for increased parent input over the next year as we engage in an overall gifted/enrichment review. This has been scheduled for 2018-19 to coincide with the instructional shifts recommended for 2018-19 to collect parent input to help us ensure we both meet the needs of all students and maintain the rigor of the core program.

11. For Additional Reading about Math: Singapore math, challenges of the new standards, importance of a growth mindset, see some of the resources below.

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