Find the Volume of a Sphere.

The volume of a sphere with a radius of $r$ units is given by the formula below.

$$V = \frac{4}{3} \pi r^3$$

Find the Surface Area of a Sphere.

From the activity, it appears that the formula for the surface area of a sphere, $S$, is:

$$S = 4\pi r^2$$

This is a result that you will prove in a later course.
Example 14 Find the volume of a sphere given its radius.

An iron ball has a diameter of 6 centimeters. Find the volume of the iron ball to the nearest cubic centimeter. Use 3.14 as an approximation for π.

\[ V = \frac{4}{3} \pi r^3 \]

\[ = \frac{4}{3} (3.14)(3^3) \]

\[ = 113.04 \]

\[ \approx 113 \text{ cm}^3 \]

The diameter of the iron ball is a segment that passes through the center and has its endpoints on the surface of the iron ball. The diameter is twice the length of the radius.
Solution

Radius of the iron ball:

\[ 6 \div 2 = 3 \text{ cm} \]

The radius is half the diameter.

Volume of the iron ball:

\[ \frac{4}{3} \pi r^3 \approx \frac{4}{3} \cdot 3.14 \cdot 3 \cdot 3 \cdot 3 \]

\[ = 113.04 \]

\[ \approx 113 \text{ cm}^3 \]

Use the formula for the volume of a sphere.

Multiply.

Round to the nearest centimeter.

The volume of the iron ball is about 113 cubic centimeters.
Guided Practice

Solve.

1. The diameter of a sphere is 8.8 meters. What is the volume of the sphere? Round your answer to the nearest hundredth. Use 3.14 as an approximation for π.

Radius of the sphere:

\[ 8.8 \div \ ? = \ ? \ m \]

The radius is half the diameter.

\[ V = \frac{4}{3} \pi r^3 \]
\[ = \frac{4}{3} \cdot (3.14) \cdot (4.4)^2 \]
\[ = \frac{4}{3} \cdot (3.14) \cdot 19.36 \]
\[ V \approx 356.64 \text{ m}^3 \]

\[ \frac{4 \cdot (267.4776)}{3} \]
\[ = \frac{1069.91104}{3} \]
\[ V = 356.627 \]
\[ \approx 356.64 \text{ m}^3 \]

Round to nearest hundredth.

\[ SA = 4 \pi r^2 \]
\[ r = 4.4 \]

\[ 4 \cdot 3.14 \cdot (4.4^2) \]
\[ 4 \cdot 3.14 \cdot 19.36 \]
\[ 243.16 \frac{1}{16} \]
\[ 243.16 \text{ m}^2 \]
Example 15  Find the radius of a sphere given its volume.

While on vacation, Mike buys a sphere made of polished agate. The volume of the sphere is 65.42 cubic inches. What is the radius of the sphere to the nearest tenth? Use 3.14 as an approximation for \( \pi \).

\[
V = \frac{4}{3} \pi r^3
\]

\[
65.42 = \frac{4}{3} (3.14) r^3
\]

\[
3 \cdot 65.42 = 12.56 r^3
\]

\[
\frac{196.26}{12.56} = \frac{12.56 r^3}{12.56}
\]

\[
r^3 = 15.62579618
\]

\[
r = 2.5 \text{ in}
\]
Solution

Volume of sphere $= \frac{4}{3} \pi r^3$

Use the formula for the volume of a sphere.

$65.42 \approx \frac{4}{3} \cdot 3.14 \cdot r^3$

Substitute for the volume and $\pi$.

$65.42 = \frac{12.56 \cdot r^3}{3}$

Multiply.

$3 \cdot 65.42 = \frac{12.56 \cdot r^3}{3} \cdot 3$

Multiply both sides by 3.

$196.26 = 12.56 \cdot r^3$

Multiply.

$\frac{196.26}{12.56} = \frac{12.56 \cdot r^3}{12.56}$

Divide both sides by 12.56.

$15.63 = r^3$

Evaluate.

$\sqrt[3]{15.63} \approx r$

Find the cube root of both sides.

$2.5 \approx r$

Round to the nearest tenth.

The radius of the sphere is about 2.5 inches.
Guided Practice

Solve.

2. Diane bought a spherical ball made of quartz at a garage sale. The volume of the ball is 1,450 cubic centimeters. What is the radius of the ball to the nearest centimeter? Use 3.14 as an approximation for π.

\[ V = \frac{4}{3} \pi r^3 \]

3. \[ 1450 = \frac{4}{3} (3.14) r^3 \]

4. \[ 1350 = 4(3.14) r^3 \]

5. \[ \frac{1350}{12.56} = r^3 \]

6. \[ 346.3375796 = r^3 \]

7. \[ r = 7 \text{ cm} \]
Find the Surface Area of a Sphere.

From the activity, it appears that the formula for the surface area of a sphere, $S$, is:

$$S = 4\pi r^2$$

This is a result that you will prove in a later course.
Example 16  Find the surface area of a sphere.

A sphere has a radius of 3 centimeters. What is the surface area of the sphere? Use 3.14 as an approximation for π.

\[
SA = 4\pi r^2 = 4(3.14)(3^2) = 113.04 \text{ cm}^2
\]
Solution  
Surface area of a sphere  
\[ = 4\pi r^2 \]  
\[ \approx 4 \cdot 3.14 \cdot 3 \cdot 3 \]  
\[ = 113.04 \text{ cm}^2 \]  
Use the formula for the surface area of a sphere.  
Substitute for \( \pi \) and \( r \).  
Evalue.

The surface area of the sphere is about 113.04 square centimeters.
Guided Practice

Solve.

3 What is the surface area of a sphere with a radius of 6 centimeters? Use 3.14 as an approximation for \( \pi \).
Example 17  

Find the radius of a sphere given its surface area.

A tennis ball has a surface area of 14,095.46 square millimeters. What is the radius of the tennis ball? Use 3.14 as an approximation for $\pi$.

\[
SA = 4\pi r^2
\]

\[
14,095.46 = 4(3.14) r^2
\]

\[
14,095.46 = 12.56 r^2
\]

\[
\frac{14,095.46}{12.56} = r^2
\]

\[
1122.25 = r^2
\]

\[
r = \sqrt{1122.25} = 33.5 \text{ mm}
\]

\[
\sqrt[2]{a} = \sqrt[3]{a^2} = \sqrt[3]{a^3}
\]

\[
\sqrt[3]{a} = \sqrt[3]{a^3}
\]

\[
\sqrt[3]{a^3} = a
\]
Solution

Surface area of tennis ball = \(4\pi r^2\)

14,095.46 = 4 \cdot 3.14 \cdot r^2
14,095.46 = 12.56 \cdot r^2
12.56 = 12.56

\[1,122.25 = r^2\]
\[\sqrt{1,122.25} = r\]
33.5 = r

Use the formula for the surface area of a sphere.
Substitute for the surface area and \(\pi\).
Multiply.
Divide both sides by 12.56.
Evaluate.
Find the square root of both sides.
Find \(r\).

The radius of the tennis ball is about 33.5 millimeters.
Guided Practice

Solve.

4. A spherical rubber ball has a surface area of 3,215.36 square centimeters. What is the radius of the rubber ball to the nearest centimeter? Use 3.14 as an approximation for $\pi$.

\[
SA = 4\pi r^2
\]
\[
3215.36 = 4(3.14)r^2
\]
\[
\frac{3215.36}{12.56} = \frac{12.56}{12.56}
\]
\[
256 = r^2
\]
\[
\boxed{r = 16 \text{ cm}}
\]