Find the Surface Area of a Cone.

The length from O to A on a right circular cone is called the slant height of the cone. The slant height is the distance between the vertex and a point on the circumference of base. When you cut the cone along $OA$ and flatten it, you will get a portion of a circle with center $O$ and radius $\ell$ units.

$$SA = \pi r^2 + \pi rl$$

Math Note

A right circular cone is a cone that has its vertex directly above the center of its base. Other cones can have the vertex off to one side, as shown.

The volume formula applies to all cones with a circular base. The surface area formula applies only to right circular cones.
Example 11 Find the area of the curved surface of a cone.

Solve. Show your work.

The diagram shows a cone with a radius of 5 centimeters and a slant height of 15 centimeters.

a) What is the exact area of the curved surface of the cone?

Solution

\[ SA = \pi r l \]

\[ = \pi (5)(15) \]

\[ = 75\pi \text{ cm}^2 \]

Use the formula for the lateral surface area of a cone.

Multiply.

The exact area of the curved surface of the cone is \(75\pi\) square centimeters.
b) What is the total surface area of the cone? Find both the exact value and an approximate value. Use 3.14 as an approximation for \( \pi \).

\[
SA = \pi r^2 + \pi rl
\]

\[
= \pi (5^2) + 75\pi
\]

\[
= 25\pi + 75\pi
\]

\[
= 100\pi \text{ cm}^2
\]

\[
\approx 314 \text{ cm}^2
\]

Area of base + Lateral surface area

\[
= \pi r^2 + 75\pi
\]

The lateral surface area is 75\( \pi \).

\[
= \pi \cdot 5 \cdot 5 + 75\pi
\]

Substitute 5 for \( r \).

\[
= 25\pi + 75\pi
\]

Multiply.

\[
= 100\pi
\]

Add.

\[
= 314 \text{ cm}^2
\]

Substitute 3.14 for \( \pi \).
11. The radius of a cone is 3 inches, and the slant height is 12 inches.

a) What is the area of the cone’s curved surface?

b) What is the total surface area of the cone? Find both the exact value and an approximate value. Use 3.14 as an approximation for \( \pi \).

\[
\begin{align*}
\text{a) } & \quad \pi rl = \pi (3)(12) \\
& = 36\pi \text{ in}^2 \\
\text{b) } & \quad \pi r^2 + \pi rl \\
& = \pi (3^2) + 36\pi \\
& = 9\pi + 36\pi \\
& = 45\pi \text{ in}^2 \\
& \approx 141.3 \text{ in}^2
\end{align*}
\]
Example 12  Find the slant height of a cone given its radius and the area of its curved surface.

The wrapper of a frozen yogurt cone has an area of 159 square centimeters. The radius of the cone is 4 centimeters. Find the slant height of the cone to the nearest tenth. Use 3.14 as an approximation for $\pi$.

Solution

Let the slant height of the cone be $l$ centimeters. Notice that the wrapper covers only the curved surface of the cone.

\[ \text{SA} = \pi rl \]
\[ 159 = 3.14(4)l \]
\[ 159 = 12.56l \]
\[ l = 12.659... \]

The slant height of the cone is about 12.7 centimeters.
Example 13  Solve a real-world problem involving a cone.

A cone-shaped roof has a radius of 14 feet and a slant height of 17 feet. The roof is covered completely with glass. The cost of glass is $40 per square foot. What is the cost of covering the roof with glass? Use 3.14 as an approximation for $\pi$.

Solution

\[
\frac{\pi rl}{2} = 3.14(14)(17) = 747.32 \text{ ft}^2
\]

\[\frac{747.32 \text{ ft}^2 \times 40}{29,892.80}
\]

Cost of covering the roof with glass:

$40 \times 747.32 = 29,892.80$

The cost of covering the roof is about $29,892.80.