6.2 Angles that Share a Vertex

Lesson Objectives
- Explore and apply the properties of angles at a point.
- Explore and apply the properties of vertical angles.

Vocabulary
- vertical angles
- congruent angles

\[ \angle A \cong \angle B \]

[Note: The text includes a diagram with a symbol indicating congruence and a note on angles being equal in measure.]
Explore and Apply the Properties of Angles at a Point.

In the diagram below, angles 1, 2, 3, and 4 share a common vertex, $O$. These angles are called angles at a point. The sum of the measures of angles at a point is $360^\circ$.

A full turn around a point is equal to $360^\circ$. 

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Example 8 Use algebra to solve problems involving angles at a point.

Find the value of $x$ in each diagram.

a)

\[
\begin{align*}
138^\circ + 84^\circ + 4x &= 360^\circ \\
222^\circ + 4x &= 360^\circ \\
4x &= 138 \\
x &= 34.5
\end{align*}
\]

Solution

\[
\begin{align*}
m\angle AOC + m\angle AOB + m\angle BOC &= 360^\circ \\
4x^\circ + 84^\circ + 138^\circ &= 360^\circ \\
4x^\circ + 222^\circ &= 360^\circ \\
4x^\circ + 222^\circ - 222^\circ &= 360^\circ - 222^\circ \\
4x &= 138 \\
\frac{4x}{4} &= \frac{138}{4} \\
x &= 34.5
\end{align*}
\]

$\angle s$ at a point
Substitute.
Simplify.
Subtract $222^\circ$ from both sides.
Simplify.
Divide both sides by 4.
Simplify.

"$\angle s$ at a point" is used as an abbreviation for angles at a point.
Solution

\[ m \angle AOC + m \angle AOB + m \angle BOC = 360^\circ \]

\[ 4x^\circ + 2x^\circ + 3x^\circ = 360^\circ \]

\[ 9x = 360 \]

\[ \frac{9x}{9} = \frac{360}{9} \]

\[ x = 40 \]

\[ \text{\(\triangle s\) at a point} \]

\[ \text{Substitute.} \]

\[ \text{Simplify.} \]

\[ \text{Divide both sides by 9.} \]

\[ \text{Simplify.} \]

Caution

Remember that the value \( x = 40 \) is not the measure of any of the angles in the diagram in b). The angle measures are as follows:

\[ m \angle AOB = 2 \cdot x^\circ = 2 \cdot 40^\circ = 80^\circ \]

\[ m \angle BOC = 3 \cdot x^\circ = 3 \cdot 40^\circ = 120^\circ \]

\[ m \angle AOC = 4 \cdot x^\circ = 4 \cdot 40^\circ = 160^\circ \]
Guided Practice

Complete.

1. Find the value of $p$ in the diagram.

\[ m_\angle AOC + m_\angle AOB + m_\angle BOC = 360^\circ \]

\[ ? + ? + ? = ? \]

\[ ? + ? = ? \]


\[ ? = ? \]

\[ ? = ? \]

\[ ? = ? \]

This is at a point

Substitute.

Simplify.

Subtract $?$ from both sides.

Simplify.

Divide both sides by $\ ?$.

Simplify.
**Example 9** Use algebra to solve problems involving angle measures.

\[ \overrightarrow{AB} = 180^\circ \]

\( \overrightarrow{AB} \) is a straight line. Find the value of each variable.

\[
\begin{align*}
4b + 80 &= 180 \\
4b &= 100 \\
b &= 25
\end{align*}
\]

\[
25 + c = 180
\]

\[
c = 155
\]

**Solution**

\[
m \angle 2 + m \angle 3 = 180^\circ
\]

Adj. \( \angle \)s on a st. line

\[
4b^\circ + 80^\circ = 180^\circ
\]

Substitute.

\[
4b^\circ + 80^\circ - 80^\circ = 180^\circ - 80^\circ
\]

Subtract 80° from both sides.

\[
4b = 100
\]

Simplify.

\[
\frac{4b}{4} = \frac{100}{4}
\]

Divide both sides by 4.

\[
b = 25
\]

Simplify.

\[
m \angle 1 + m \angle 4 = 180^\circ
\]

Adj. \( \angle \)s on a st. line

\[
c^\circ + b^\circ = 180^\circ
\]

Substitute.

\[
c^\circ + 25^\circ = 180^\circ
\]

Substitute \( b = 25 \).

\[
c^\circ + 25^\circ - 25^\circ = 180^\circ - 25^\circ
\]

Subtract 25° from both sides.

\[
c = 155
\]

Simplify.
Guided Practice

Solve.

2. \( \overline{BE} \) and \( \overline{CA} \) are straight lines. Find the value of each variable.

\[
\begin{align*}
q + 152 &= 180 \\
q &= 28 \\
\frac{r + 152}{1} &= 180 \\
r &= 28
\end{align*}
\]

\( \angle AOB + \angle AOE = 180^\circ \)

\[
\begin{align*}
? + ? &= ? \\
? &= ?
\end{align*}
\]

adj. \( \angle \)'s on a st. line
Substitute.
Subtract 152° from both sides.
Simplify.

\( \angle BOC + \angle COD + \angle DOE = 180^\circ \)

\[
\begin{align*}
? + ? &= ? \\
? &= ? \\
\frac{?}{?} &= \frac{?}{?} \\
? &= ?
\end{align*}
\]

adj. \( \angle \)'s on a st. line
Substitute.
Simplify.
Subtract 68° from both sides.
Simplify.
Divide both sides by ?.
Simplify.
3 \overrightarrow{PQ} is a straight line. Find the value of each variable.
Example 10 Use ratios to find angle measures in a diagram.

In the diagram, the ratio $a : b : c = 1 : 2 : 2$. Find the values of $a$, $b$, and $c$.

\[ x + 2x + 2x = 360 \]
\[ 5x = 360 \]
\[ x = 72 \]
\[ a = 72 \]
\[ b = 144 \]
\[ c = 144 \]

Solution

Method 1

Use bar models.

\[ a° + b° + c° = 360° \]
\[ \text{\angle s at a point} \]

Since $a : b : c = 1 : 2 : 2$, if you represent $a$ by 1 unit, then $b$ and $c$ are 2 units each in the bar model. The total number of units in the bar model is $1 + 2 + 2 = 5$ units.

5 units $\rightarrow$ 360

1 unit $\rightarrow$ $\frac{360}{5} = 72$

$a = 72$

$b = 2 \cdot 72 = 144$

$c = 2 \cdot 72 = 144$
Method 2

Use a variable to represent the measure of the angle.

The ratio \( a : b : c = 1 : 2 : 2 \). So, \( b = 2 \cdot a \) and \( c = 2 \cdot a \).

\[
\begin{align*}
a^\circ + b^\circ + c^\circ &= 360^\circ & \triangle \text{ at a point} \\
a + 2a + 2a &= 360 & \text{Substitute.} \\
5a &= 360 & \text{Simplify.} \\
\frac{5a}{5} &= \frac{360}{5} & \text{Divide both sides by 5.} \\
a &= 72 & \text{Simplify.}
\end{align*}
\]

\[
\begin{align*}
b &= 2 \cdot a \\
&= 2 \cdot 72 & \text{Substitute } a = 72.
\end{align*}
\]

\[
\begin{align*}
c &= 2 \cdot a \\
&= 2 \cdot 72 & \text{Substitute } a = 72.
\end{align*}
\]

\[
\begin{align*}
&= 144 & \text{Simplify.}
\end{align*}
\]
**Guided Practice**

Complete.

4. In the diagram at the right, the ratio \( a : b : c = 1 : 3 : 5 \).
Find the values of \( a, b, \) and \( c \).

The ratio \( a : b : c = 1 : 3 : 5 \). So, \( b = 3 \cdot a \) and \( c = 5 \cdot a \).

\[
\quad + \quad + \quad = \quad \quad \quad \angle s \text{ at a point}
\]

\[
\quad = \quad \quad \quad \text{Substitute.}
\]

\[
\quad = \quad \quad \quad \text{Simplify.}
\]

\[
\quad \div \quad \quad \quad \text{Divide both sides by} \quad \quad \quad \text{Simplify.}
\]

\[ b = 3 \cdot a \]

\[ = \quad \quad \quad \quad \text{Substitute} \quad a = \quad \quad \quad \text{Substitute} \quad a = \quad \quad \quad \text{Simplify.}
\]

\[ c = 5 \cdot a \]

\[ = \quad \quad \quad \quad \text{Substitute} \quad a = \quad \quad \quad \text{Simplify.}
\]
Explore and Apply the Properties of \textbf{Vertical Angles}.

When two lines intersect each other at a point, they form four angles. The nonadjacent angles are called vertical angles.

In the diagram below, $\angle 1$ and $\angle 3$ are vertical angles, and $\angle 2$ and $\angle 4$ are vertical angles.

Because $\angle 1$ and $\angle 2$ are adjacent angles that form a straight line, they are supplementary. $\angle 2$ and $\angle 3$ are also adjacent angles that form a straight line. So, they are also supplementary.

You can deduce that $m\angle 1 + m\angle 2 = 180^\circ$ and $m\angle 2 + m\angle 3 = 180^\circ$.

From the equations, you can see that $\angle 1$ and $\angle 3$ are equal in measure. When two angles have the same angle measure, they are called \textbf{congruent angles}. So, vertical angles are congruent.
Example 11  Use algebra to solve problems involving vertical angles.

\( \overline{AB} \) and \( \overline{CD} \) are straight lines. Find the value of \( x \).

\[
\begin{align*}
2x^\circ &= 60^\circ \\
\frac{2x}{2} &= \frac{60}{2} \\
x &= 30
\end{align*}
\]

“Vert. \( \angle s \)” is used as an abbreviation for vertical angles.
**Guided Practice**

Complete.

5. \(\overline{AB}\) and \(\overline{CD}\) are straight lines. Find the value of \(y\).

\[
\frac{?}{?} = \frac{?}{?} \quad \text{Vert. } \angle \text{s}
\]

\[
\frac{?}{?} = \frac{?}{?} \quad \text{Divide both sides by } ?.
\]

\[
? = ? \quad \text{Simplify.}
\]
Example 12  Apply reasoning to find measures of angles formed by intersecting lines.

In the diagram, two straight lines intersect to form angles 1, 2, 3, and 4. Find the value of each variable if $m\angle 1 = 76^\circ$.

Solution

$m\angle 1 + m\angle 2 = 180^\circ$
$76^\circ + a^\circ = 180^\circ$
$76^\circ + a^\circ - 76^\circ = 180^\circ - 76^\circ$
$a = 104$  

$\angle 3 = \angle 1$  
$b^\circ = 76^\circ$  
$b = 76$

$\angle 4 = \angle 2$  
$c^\circ = 104^\circ$  
$c = 104$  

180$-76$
76$\angle$
80$-76$
360$-2(76)$
2