Lesson 5.3

Lesson Objective
- Solve real-world direct proportion problems.

Vocabulary
- cross products
Solve Real-World Direct Proportion Problems.

Proportion: a pair of equal ratios

Because the ratios $\frac{2}{5}$ and $\frac{4}{10}$ are equivalent, you can use them to write a proportion. Notice what happens when you find the cross products of the proportion.

Cross products:
$2 \cdot 10 = 4 \cdot 5$

Multiply the numerator of the left fraction by the denominator of the right fraction. Multiply the numerator of the right fraction by the denominator of the left fraction.

$20 = 20$

Simplify.

If you find the cross products of other proportions, you will see that the cross products of a proportion are always equal.
Cross products property:
If \( \frac{a}{b} = \frac{c}{d} \), where \( b \neq 0 \) and \( d \neq 0 \), then \( ad = bc \).

You can use the cross products property to solve problems that involve quantities that are in direct proportion.

For example, suppose you pay $40 for 8 T-shirts at a store. Given that the cost of T-shirts is directly proportional to the number of T-shirts you buy, you can use proportionality reasoning to find the cost of 5 T-shirts.

You can use a proportion, or you can use a direct proportion equation.

\[
\begin{align*}
\frac{40}{8 \text{ shirts}} &= \frac{x}{5 \text{ shirts}} \\
40 \times 5 &= 8 \times x \\
200 &= 8x \\
\frac{200}{8} &= x \\
25 &= x
\end{align*}
\]

\[
\begin{align*}
k &= \frac{y}{x} \\
k &= \frac{40}{8} \\
k &= 5 \\
\text{cost of one T-shirt} &= 5 \text{ dollars} \\
y &= 5x \\
y &= 5(25) \\
y &= 125
\end{align*}
\]
Example 8  Solve a real-world direct proportion problem.

Belle works at a convenience store. The amount of money she earns is directly proportional to the number of hours she works. She is paid $432 for 18 hours of work. Use a proportion to find how much Belle is paid for 21 hours of work.

\[
K = \frac{X}{18} \quad \text{she makes } \$24/\text{hr.}
\]

\[
K = \frac{24}{18} \quad \Rightarrow \quad X = 24 \cdot (21)
\]

\[
y = 504
\]

Solution

Method 1

Use a proportion.

Let \( y \) be the amount of money Belle is paid.

\[
\frac{432 \text{ dollars}}{18 \text{ h}} = \frac{y \text{ dollars}}{21 \text{ h}}
\]

\[
\frac{432}{18} = \frac{y}{21}
\]

\[
y \cdot 18 = 21 \cdot 432
\]

\[
y = 9,072
\]

\[
\frac{18y}{18} = \frac{9,072}{18}
\]

\[
y = 504
\]

Belle is paid $504 for 21 hours of work.
\[
\frac{432}{18h} = \frac{x}{21h}
\]
\[
21 \times 432 = 18 \times x
\]
\[
\frac{x}{21h} = \frac{432}{18h}
\]
\[
21 \times 432 = x \times 18
\]
\[
\frac{18h}{432} = \frac{x}{21h}
\]
\[
18 \times x = 432 \times 21
\]
Guided Practice

Solve.

1. At a factory, the number of cars produced is directly proportional to the number of hours factory workers are making the cars. It takes 45 hours to make 60 cars. Use a proportion to find how long it will take to make 250 cars.

   \[
   \frac{60}{45} = \frac{250}{x}
   \]

   \[
   60x = 45 \cdot 250
   \]

   \[
   60x = 11,250
   \]

   \[
   \frac{45}{60} = \frac{x}{250}
   \]

   \[
   \frac{60x}{60} = \frac{11,250}{60}
   \]

   \[
   x = \frac{11,250}{60}
   \]

   \[
   x = 187.5
   \]

   \[
   187.5 \text{ hrs}
   \]

   It takes \(\frac{187.5}{60}\) hours to make 250 cars.

Method 1

Use a proportion.

Let \(x\) be the number of hours it takes to make 250 cars.

\[
\frac{60 \text{ cars}}{? \text{ hours}} = \frac{? \text{ cars}}{x \text{ hours}}
\]

\[
x \cdot 60 = ? \cdot ?
\]

\[
60x = ?
\]

\[
\frac{60x}{?} = \frac{?}{?}
\]

\[
x = ?
\]

Write a proportion.

Write ratios as fractions.

Write cross products.

Simplify.

Divide both sides by ?.

Simplify.