Representing Direct Proportion Graphically

Lesson Objective

- Use a graph to interpret direct proportion.
Use a Graph to Interpret Direct Proportion.

Each time the wheel on Mike’s unicycle goes around, the unicycle moves forward 2 meters. The distance the unicycle moves forward is directly proportional to the number of revolutions.

The table and the graph show the relationship between the number of revolutions and distance the wheel moves.

<table>
<thead>
<tr>
<th>Revolutions ((x))</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance ((y \text{ meters}))</td>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

\[
\frac{\text{Distance}}{\text{Revolutions}} = \frac{2}{1} = \frac{4}{2} = \frac{6}{3} = 2
\]

The graph of a direct proportion is always a straight line through the origin, \((0, 0)\), that does not lie along the horizontal or vertical axis.
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For the graph above, each point \((x, y)\) means that in \(x\) revolutions, the unicycle wheel moves \(y\) meters. For example, the point \((0, 0)\) means that in 0 revolution, the wheel moves 0 meter. The point \((1, 2)\) means that in 1 revolution, it moves 2 meters.

The point \((1, 2)\) can be used to find the constant of proportionality: \(\frac{2}{1} = 2\)

**Think Math**

Because \(k = \frac{y}{x}\), any point \((x, y)\) except \((0, 0)\) on the line of a direct proportion can be used to find \(k\). Why is the point \((1, y)\) a convenient point to use?
Example 6 Identify direct proportion from a graph.

Tell whether each graph represents a direct proportion. If so, find the constant of proportionality. Then write a direct proportion equation.

a)

\[ k = 10 \]
\[ y = 10x \]

Solution

The graph is a straight line through the origin, and it does not lie along the x- or y-axis. So, it represents a direct proportion.

Because the graph passes through \((1, 10)\), the constant of proportionality is 10.

The direct proportion equation is \(y = 10x\).
Solution

Although the graph is a straight line that does not lie along the x- or y-axis, it does not pass through the origin. So, the graph does not represent a direct proportion.
Solution

Although the graph passes through the origin and does not lie along the x- or y-axis, it is not a straight line. So, it does not represent a direct proportion.
Example 7  Interpret a graph of direct proportion.

Jonathan works at a bookstore. The amount of money he earns is directly proportional to the number of hours he works. The graph shows the amount of money, \( w \) dollars, he earns in \( t \) hours.

a) Find the constant of proportionality. How much does Jonathan earn per hour?
\[ K = \frac{15}{1} = \$15/\text{hr}. \]

b) Write a direct proportion equation.
\[ w = 15t \]

c) Explain what the point \((2, 30)\) represents in this situation.
He makes $30 in 2 hrs.

d) If Jonathan works 4 hours, how much will he earn?
\[ \text{graph} \to (4, 60) \]
\[ \text{equation} \to w = 15t = 15(4) \]
\[ w = 60 \]
Solution

Because the graph passes through $(1, 15)$, the constant of proportionality is $15$.

In this case, the constant of proportionality is the amount of money earned per hour. So, Jonathan earns money at a rate of $15$ per hour.
b) Write a direct proportion equation.

Solution
The direct proportion equation is $w = 15t$.

c) Explain what the point (2, 30) represents in this situation.

Solution
It means that Jonathan earns $30 in 2 hours.

d) If Jonathan works 4 hours, how much will he earn?

Solution
From the graph, Jonathan will earn $60 in 4 hours.

e) If Jonathan wants to earn $45, how long should he work?

Solution
From the graph, Jonathan should work for 3 hours.

Think Math
How can you use the graph to find the number of hours Jonathan should work if he wants to earn $65?
Guided Practice
Complete.

Ms. Gray is driving on a long distance trip. The distance she travels is directly proportional to time she travels. The graph shows the distance she travels, \( y \) miles, after \( t \) hours.

\[ k = 50 \]
\[ \text{she travels} \]
\[ 50 \text{ mph} \]

\( y = 50t \)
Use graph paper. Solve.

8 Beth works at a pottery studio. She is making ceramic pots to sell at a craft fair. Graph the relationship between the number of ceramic pots she makes, \( y \), and the number of days she works at the studio, \( x \). Use 1 unit on the horizontal axis to represent 1 day and 1 unit on the vertical axis to represent 5 ceramic pots.

<table>
<thead>
<tr>
<th>Number of Days (( x ))</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Pots (( y ))</td>
<td>0</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
<td>30</td>
</tr>
</tbody>
</table>

a) Determine whether the graph represents a direct proportion. If so, find the constant of proportionality and write the direct proportion equation.

b) Explain what the point (4, 20) represents in this situation.

c) How many pots can Beth make in 3 days?

d) Beth will not start selling pots until she has made at least 30. How long will it take her to make that many pots?