Identify Direct Proportion from an Equation.

When $y$ is directly proportional to $x$, you can write $\frac{y}{x} = k$. You can use algebra to write another form of this equation, giving $y$ in terms of $x$. For example, when $k = 2$, you can multiply both sides of the equation $\frac{y}{x} = 2$ by $x$ to get the equivalent equation $y = 2x$.

\[
\frac{y}{x} = 2 \quad \text{Write an equation.}
\]
\[
x \cdot \frac{y}{x} = x \cdot 2 \quad \text{Multiply both sides by } x \text{ to undo the division of } y \text{ by } x.
\]
\[
y = 2x \quad \text{Simplify.}
\]

You can also use algebra to decide if an equation represents a direct proportion.

Think Math

In the equation $y = 2x$, $x$ represents pounds of strawberries, and $y$ represents the cost of strawberries. How can you use the equation to find the cost of buying 10 pounds of strawberries?
Example 2  Tell whether quantities are in direct proportion from an equation.

Tell whether each equation represents a direct proportion. If so, identify the constant of proportionality.

\[ a ) \frac{1}{2} y = 3x \cdot 2 \]

\[ y = 6x \]

Yes! COP \( k = 6 \)

Solution

\[ \frac{1}{2} y = 3x \]

\[ 2 \cdot \frac{1}{2} y = 2 \cdot 3x \]

\[ y = 6x \]

Multiply both sides by 2.

Simplify.

Because the original equation \( \frac{1}{2} y = 3x \) can be rewritten as an equivalent equation in the form \( y = kx \), it represents a direct proportion. The constant of proportionality is 6.
Guided Practice

Tell whether each equation represents a direct proportion. If so, identify the constant of proportionality.

3. \(0.4y = x\)

\[
\begin{align*}
0.4y &= x \\
\frac{0.4y}{x} &= \frac{x}{x} \\
y &= \text{?}
\end{align*}
\]

Divide both sides by \(x\).

Simplify.

Because the original equation \(0.4y = x\) can be rewritten as an equivalent equation in the form \(y = kx\), it is a direct proportion. The constant of proportionality is \(\frac{5}{2}\).
Tell whether each equation represents a direct proportion. If so, find the constant of proportionality.

4 \[ x = 1 - 2y \]

\[
\begin{align*}
x &= 1 - 2y \\
x + 2y &= 1 - 2y + 2y \\
x + 2y - \ ? &= 1 - \ ? \\
2y &= 1 - \ ? \\
\frac{2y}{?} &= \frac{1}{?} \\
y &= \ ?
\end{align*}
\]

Add 2y to both sides.

Subtract \(?\) from both sides.

Simplify.

Divide both sides by \(?\).

Simplify.

Because the original equation \( x = 1 - 2y \) ___, be rewritten as an equivalent equation in the form \( y = kx \), it ___ a direct proportion.

Think Math

Adam says the equation \( 5y + 2y = 7 \) represents a direct proportion. Susan disagrees with him. Who is correct?
Recognize that a Constant of Proportionality can be a Unit Rate.

The constant of proportionality in a direct proportion often represents a unit rate. For instance, in the example about buying strawberries, the constant of proportionality 2 represents the unit cost of the strawberries. The total cost of the strawberries, \( y \), is the product of the unit cost and the weight of the strawberries purchased, \( x \) pounds.

So, the equation of the direct proportion is: \( y \) dollars = \( \frac{\$2}{1 \text{ pound}} \) \cdot x \text{ pounds}

\[ y = 2x \]
Example 3  Identify a constant of proportionality from a table.

The table shows the price, $P$ dollars, for $x$ cans of soup. $P$ is directly proportional to $x$.

Find the constant of proportionality and tell what it represents in this situation. Then write a direct proportion equation.

<table>
<thead>
<tr>
<th>Number of Cans ($x$)</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price ($P$ dollars)</td>
<td>1.60</td>
<td>3.20</td>
<td>4.80</td>
</tr>
</tbody>
</table>

$k = 1.60$ represents the cost of 1 can of soup

$p = k \cdot x$

$p = 1.60 \cdot x$

Solution

Constant of proportionality: $\frac{1.60}{1 \text{ can}} = 1.6$

The constant of proportionality is 1.6 and represents the cost, in dollars, per can of soup. The direct proportion equation is $P = 1.6x$. 
Guided Practice
Copy and complete.

5 The table shows the number of baseballs, \( y \), made in \( x \) days. The number of baseballs made is directly proportional to the number of days of production. Find the constant of proportionality and tell what it represents in this situation. Then write a direct proportion equation.

<table>
<thead>
<tr>
<th>Number of Days (( x ))</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Baseballs (( y ))</td>
<td>56</td>
<td>112</td>
<td>168</td>
</tr>
</tbody>
</table>

Constant of proportionality: __?
The constant of proportionality is __ and represents __.
The direct proportion equation is __.

\( K = 56 \)
\# of balls made per day
\( y = 56x \)
Example 4  Identify a constant of proportionality in a verbal description.

Alina is buying some baseball caps. Each cap costs $8. The amount Alina pays for the caps is directly proportional to the number of caps she buys. Write an equation that represents the direct proportion.

\[ \eta = \# \text{ of caps} \]
\[ C = \text{total cost} \]
\[ C = 8 \eta \]

Solution

Let \( x \) be the number of baseball caps Alina buys.
Let \( y \) be the amount she pays.

Cost per baseball cap: $8 per cap.

The direct proportion equation is \( y = 8x \).
Guided Practice
Copy and complete.

A cafeteria sells sandwiches for $4 each. The amount Jason pays for some sandwiches is directly proportional to the number he buys. Write an equation that represents the direct proportion.

Let $\_\_\_$ be the number of sandwiches.
Let $\_\_\_$ be the amount Jason pays.

Cost per sandwich: $\_\_\_\_\$ per sandwich

The direct proportion equation is $\_\_\_\_\_\_ = \_\_\_\_\$. 
Example 5  Identify the constant of proportionality in an equation.

Solve. Show your work.

\( y \) is directly proportional to \( x \), and \( y = 3 \) when \( x = 9 \). Find the constant of proportionality. Then write a direct proportion equation.

Since \( y \) is directly proportional to \( x \), you can use \( \frac{y}{x} = k \) to find the constant of proportionality, \( k \).

\[
\frac{y}{x} = k \quad \frac{3}{9} = k \quad \frac{1}{3} = k
\]

\[y = \frac{1}{3} x\]

Solution

Constant of proportionality: \( \frac{y}{x} = \frac{3}{9} \)

\[= \frac{1}{3} \quad \text{Write in simplest form.}\]

The constant of proportionality is \( \frac{1}{3} \).

The direct proportion equation is \( y = \frac{1}{3} x \).
Guided Practice

Copy and complete.

7. \( q \) is directly proportional to \( p \), and \( p = 12 \) when \( q = 24 \). Find the constant of proportionality. Then write a direct proportion equation.

\[ q = 2p \]

\[ K = \frac{24}{12} = 2 \]

Constant of proportionality: \( \frac{q}{p} = ? \)

\[ = ? \]

Write in simplest form.

The constant of proportionality is ___.
The direct proportion equation is ___.

Solve.

8. \( w \) is directly proportional to \( h \), and \( w = 18 \) when \( h = 3 \). Find the constant of proportionality. Then write a direct proportion equation.