Lesson 3.4

**Expanding Algebraic Expressions**

**Lesson Objective**
- Expand algebraic expressions with fractional, decimal, and negative factors.

\[
\begin{align*}
\frac{6m}{5(a+2)} &= \frac{5a+5\cdot2}{5a+10} \\
\end{align*}
\]

Distributive Property:

\[
\begin{align*}
a(b+c) &= a\cdot b + a\cdot c \\
a(b-c) &= a\cdot b - a\cdot c \\
\end{align*}
\]
\[ \frac{7x + 5x - 9}{12x + 9} \quad \rightarrow \quad \frac{7x + 5x + 9}{12x - 9} \]
**Expand Algebraic Expressions with Fractional Factors.**

You have learned how to expand algebraic expressions involving integers, like $3(2x + 4)$ and $2(5x - 1)$. You use the distributive property to expand such expressions.

$$3(2x + 4) = 3(2x) + 3(4)$$
$$= 6x + 12 \quad \text{Use the distributive property. Multiply.}$$

$$2(5x - 1) = 2(5x) - 2(1)$$
$$= 10x - 2 \quad \text{Use the distributive property. Multiply.}$$

You obtain an expression equivalent to the original expression after expanding.

$3(2x + 4)$ and $6x + 12$ are equivalent expressions because the value of both expressions remains the same for all values of $x$. In the same way, $2(5x - 1)$ and $10x - 2$ are also equivalent expressions.

You can expand algebraic expressions like $\frac{1}{2}(2x + 4)$ using either bar models or the distributive property. This will produce an equivalent expression, just as expanding an expression with a whole number factor did.
**Example 10**  Expand algebraic expressions with fractional factors.

Expand the expression $\frac{1}{3}(3x + 15)$.

$$\frac{1}{3} \cdot 3x + \frac{1}{3} \cdot 15 = \frac{1}{3} \cdot 3x + \frac{1}{3} \cdot 15\frac{x + 5}{x + 5}$$

**Solution**

**Method 1**

<table>
<thead>
<tr>
<th>x</th>
<th>5</th>
<th>x</th>
<th>5</th>
<th>x</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{3}(3x + 15)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From the bar model,

$\frac{1}{3}(3x + 15) = x + 5$

**Guided Practice**

Copy and complete to expand the expression.

1. **Method 1**

<table>
<thead>
<tr>
<th>2x</th>
<th>3</th>
<th>?</th>
<th>?</th>
<th>?</th>
<th>?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{4}(8x + 12)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From the bar model, $\frac{1}{4}(8x + 12) =$ ?

2. **Method 2**

$$\frac{1}{4}(3x + 15) = \frac{1}{4}(3x) + \frac{1}{4}(15)$$

Use the distributive property.

$\frac{1}{4}(3x) + \frac{1}{4}(15) = x + 5$

Multiply.
Expand each expression.

2 \[ \frac{1}{3} (9x + 6) \]
\[ = \frac{1}{3} \cdot 9x + \frac{1}{3} \cdot 6 \]
\[ = 3x + 2 \]

3 \[ \frac{1}{5} (25x + 15) \]
\[ = \frac{1}{5} \cdot 25x + \frac{1}{5} \cdot 15 \]
\[ = 5x + 3 \]
Expand Algebraic Expressions with Decimal Factors.

You can also use the distributive property to expand expressions involving decimals, such as $0.2(4x + 3)$.

\[ 0.2(4x + 3) = 0.2(4x) + 0.2(3) \]
\[ = 0.8x + 0.6 \]

Use the distributive property.
Multiply.

**Example 11** Expand algebraic expressions with decimal factors.

Expand the expression $0.7(0.2t - 3)$.

**Solution**

\[ 0.7(0.2t - 3) = 0.7(0.2t + (-3)) \]
Rewrite subtraction as a sum.
\[ = 0.7(0.2t) + 0.7(-3) \]
Use the distributive property.
\[ = 0.14t + (-2.1) \]
Multiply.
\[ = 0.14t - 2.1 \]
Rewrite the expression.

**Caution**
Writing subtraction as the sum of a negative number will help you work more carefully and not lose track of negative signs.
Guided Practice
Copy and complete to expand each expression. Write + or − in each ?.

4. \(0.3(2x + 5) = 0.6x + 1.5\)
   \[0.3(2x + 5) = 0.3(?_\_) + 0.3(?_\_)
   = \_? + \_?
\]

5. \(0.5(1.4y - 2.1) = 0.7y - 1.05\)
   \[0.5(1.4y - 2.1) = 0.5(?_\_) \_? 0.5(- ?_\_)
   = \_? \_? (-?_\_)
   = \_? \_? \_?
\]

Expand each expression.

6. \(0.4(3y + 2) = 1.2y + 0.8\)

7. \(0.2(4x - 3.1) = 0.8x - 0.62\)