2.5 Operations with Rational Numbers

Lesson Objectives
- Add and subtract rational numbers.
- Multiply and divide rational numbers.

Vocabulary
- complex fraction
- least common denominator
Add Rational Numbers.

A rational number is a number that can be written as \( \frac{m}{n} \), where \( m \) and \( n \) are integers with \( n \neq 0 \).

If \( \frac{m}{n} \) is negative, either \( m \) or \( n \) can be negative but not both \( m \) and \( n \).

You may rewrite rational numbers with a common denominator before you add them.

**Math Note**

If \( \frac{m}{n} \) is any rational number, then \( -\frac{m}{n} = \frac{-m}{n} = \frac{m}{-n} \).

For example:

\[
\begin{align*}
-\frac{7}{15} + \frac{7}{15} &= \frac{7}{-15} \\
&= \frac{7}{-15} 
\end{align*}
\]

**The sum of two rational numbers with the same denominator:**

Let \( \frac{a}{b} \) and \( \frac{c}{b} \) be any rational numbers.

\[
\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}
\]

**The sum of two rational numbers with different denominators:**

Let \( \frac{a}{b} \) and \( \frac{c}{d} \) be any rational numbers.

\[
\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}
\]
To add two rational numbers, such as $\frac{1}{5}$ and $\frac{-2}{3}$, you can apply the rules you know for adding integers.

First find the **least common denominator** (LCD) of $\frac{1}{5}$ and $\frac{-2}{3}$, which is 15. Then use the LCD to write equivalent fractions and evaluate.

\[
\frac{1}{5} + \frac{-2}{3}
\]

\[
\frac{3}{15} + \frac{-10}{15} = \frac{3+(-10)}{15} = \frac{-7}{15}
\]
Example 14  Add rational numbers.

Evaluate each expression.

a) $\frac{-2}{7} + \frac{3}{-5}$

\[ \frac{-2}{7} + \frac{-3}{5} \]

\[ \frac{-10}{35} + \frac{-21}{35} = \frac{-31}{35} \]
b) \[ \frac{1}{3} + \left(-2\frac{1}{6}\right) \]

\[ \frac{4}{6} + \frac{-13}{6} = \frac{-5}{6} \]

\[ 1 + \frac{1}{3} + (-2) + \left(-\frac{1}{6}\right) \]

\[ \frac{1}{3} + \frac{-2}{6} \]

\[ \frac{2}{6} + \frac{-2}{6} = -1 + \frac{1}{6} = \frac{-5}{6} \]

**Caution**

The mixed number \(-2\frac{1}{6}\) is the sum of \(-2\) and \(-\frac{1}{6}\). Both the fraction part and the integer part of the mixed number are negative.

\[-2\frac{1}{6} = -2 + \left(-\frac{1}{6}\right) = -2 - \frac{1}{6} \]

\[-2\frac{1}{6} \neq -2 + \frac{1}{6} \]
\[
\begin{align*}
3 \quad & \frac{1}{6} + \left( \frac{-5}{9} \right) + \left( \frac{-1}{3} \right) \\
& \frac{3}{18} + \left( \frac{-10}{18} \right) + \left( \frac{-16}{18} \right) \\
& \frac{3}{18} + -\frac{16}{18} = \frac{-13}{18}
\end{align*}
\]

**Solution**

**Method 1**

Add two rational numbers at a time, working from left to right.

\[
\frac{2}{5} + \left( \frac{-4}{15} \right) + \frac{1}{10} = \frac{2 \cdot 3}{5 \cdot 3} + \frac{-4}{15} + \frac{1}{10}
\]

Write equivalent fractions for the first two fractions using the LCD.

Multiply all products.

Write the first two fractions using a single denominator.

Add.

Write equivalent fractions using the LCD of the fractions in the new sum.

Multiply all products.

Add the like fractions.
Guided Practice

Copy and complete.

1. $\frac{-1}{9} + \frac{2}{-5}$
   
   $\frac{-1}{9} + \frac{-2}{5}$
   
   $\frac{-5 + -18}{45}$
   
   $= \frac{-23}{45}$

2. $-1\frac{1}{6} + 3\frac{4}{9} = 2\frac{5}{18}$
   
   $-\frac{7}{6} + \frac{31}{9}$
   
   $-\frac{21}{18} + \frac{62}{18}$
   
   $= \frac{41}{18} = 2\frac{5}{18}$

3. $\frac{1}{6} + \left(-\frac{5}{9}\right) + \left(-\frac{1}{3}\right)$

   Same as Example!
Subtract Rational Numbers.

As with fractions, you may need to rewrite rational numbers so that they have a common denominator before you subtract.

The difference of two rational numbers with the same denominator:

Let \( \frac{a}{b} \) and \( \frac{c}{b} \) be any rational numbers.

\[
\frac{a}{b} - \frac{c}{b} = \frac{a - c}{b}
\]

The difference of two rational numbers with different denominators:

Let \( \frac{a}{b} \) and \( \frac{c}{d} \) be any rational numbers.

\[
\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}
\]

\[-3 - 8\]
\[-3 + -8\]
\[-\frac{3}{5} - \frac{8}{12}\]
\[-\frac{3}{5} + -\frac{8}{12}\]
Example 15  Subtract rational numbers.

Evaluate each expression.

a) \[ \frac{1}{6} - \frac{3}{4} = \frac{-7}{12} \]
b) \[ \frac{2\frac{1}{4}}{4} - 4\frac{2}{3} = \frac{9}{4} - \frac{14}{3} \]
\[ = \frac{27}{12} - \frac{56}{12} \]
\[ = \frac{-29}{12} = -2\frac{5}{12} \]
c) \[
\frac{1}{3} - \frac{11}{12} - \frac{1}{2} = \frac{4}{12} - \frac{11}{12} - \frac{6}{12}
\]

\[
-\frac{7}{12} - \frac{6}{12}
\]

\[
-\frac{7}{12} + \frac{6}{12}
\]

\[
-\frac{13}{12} = -1\frac{1}{12}
\]
Guided Practice
Evaluate each expression.

4. \( \frac{1}{4} - \frac{3}{10} = \frac{-1}{20} \)

5. \( \frac{7}{8} - \frac{9}{10} = \frac{-1}{40} \)
6 \quad \frac{3 \frac{1}{4}}{4} - \frac{7 \frac{5}{6}}{6} \\
\frac{13}{4} - \frac{47}{6} \\
\frac{39}{12} - \frac{94}{12} \\
-\frac{55}{12} = \boxed{-4 \frac{7}{12}}

7 \quad \frac{3}{7} - \frac{27}{28} - \frac{3}{14} \\
\frac{12}{28} - \frac{27}{28} - \frac{6}{28} \\
-\frac{15}{28} - \frac{6}{28} \\
-\frac{21}{28} = \boxed{-\frac{3}{4}}