Lesson 5.4  Understanding Inverse Proportion

Tell whether two quantities are in inverse proportion. If so, find the constant of proportionality.

1. \[
\begin{array}{cccc}
\text{x} & 100 & 50 & 10 \\
\text{y} & 2 & 4 & 20
\end{array}
\]
\[
2 \cdot 100 = 4 \cdot 50 = 2 \cdot 10 \\
K = 200
\]

2. \[
\begin{array}{cccc}
\text{x} & 6 & 4 & 2 \\
\text{y} & 20 & 40 & 80
\end{array}
\]
\[
20 \cdot 6 \neq 40 \cdot 4 \\
\text{No}
\]
\[
xy = K \\
y = \frac{K}{x}
\]

3. \[
\begin{array}{cccc}
\text{x} & 3 & 6 & 9 \\
\text{y} & 10 & 20 & 30
\end{array}
\]
\[
10 \cdot 3 \neq 20 \cdot 6 \\
\text{No}
\]

4. \[
\begin{array}{cccc}
\text{x} & 2 & 6 & 10 \\
\text{y} & 210 & 70 & 42
\end{array}
\]
\[
210 \cdot 2 = 70 \cdot 6 = 42 \cdot 10 \\
K = 420
\]

Tell whether each equation represents an inverse proportion. If so, give the constant of proportionality.

5. \[y = 15x\]
\[\text{NO! (direct prop)}\]

6. \[xy = \frac{1}{4}\]
\[\frac{xy}{x} = \frac{1}{4}\]
\[y = \frac{1}{4} \div x = \frac{1}{4} \cdot \frac{1}{x} \Rightarrow y = \frac{K}{x}\]

7. \[20y = \frac{4}{5}\]
\[y = \frac{4}{20} \cdot \frac{1}{x} = \frac{1}{x} \cdot 0.2 \Rightarrow y = \frac{1}{5x} \text{ OR } y = \frac{0.2}{x}\]

8. \[xy + 3 = 7\]
\[xy = 4\]
\[y = \frac{4}{x}\]

\[\text{Yes!} \Rightarrow K = 4\]
\[\text{Yes!} \Rightarrow K = 0.2 \text{ OR } \frac{1}{5}\]
Each graph represents an inverse proportion. Find the constant of proportionality.

9. \( y \)  \[
\begin{array}{|c|c|c|}
\hline
x & y \\
\hline
0 & 30 \\
1 & 20 \\
2 & 10 \\
3 & 0 \\
\hline
\end{array}
\]
\( (1, 10) \)

The constant of proportionality is \( 10 \). (x,y)

10. \( y \)  \[
\begin{array}{|c|c|c|}
\hline
x & y \\
\hline
0 & 15 \\
1 & 10 \\
2 & 5 \\
3 & 0 \\
\hline
\end{array}
\]
\( (3, 5) \)

The constant of proportionality is \( 15 \).

11. \( y \)  \[
\begin{array}{|c|c|c|}
\hline
x & y \\
\hline
0 & 0.6 \\
1 & 0.4 \\
2 & 0.2 \\
3 & 0 \\
\hline
\end{array}
\]
\( (2, 0.2) \)

The constant of proportionality is \( 0.4 \).

12. \( y \)  \[
\begin{array}{|c|c|c|}
\hline
x & y \\
\hline
0 & 8 \\
1 & 4 \\
2 & 2 \\
3 & 1 \\
\hline
\end{array}
\]
\( (2, 4) \)

The constant of proportionality is \( 8 \).

13. \( y \)  \[
\begin{array}{|c|c|c|}
\hline
x & y \\
\hline
0 & 150 \\
2 & 75 \\
4 & 30 \\
6 & 0 \\
\hline
\end{array}
\]
\( (4, 50) \)

The constant of proportionality is \( 200 \).

14. \( y \)  \[
\begin{array}{|c|c|c|}
\hline
x & y \\
\hline
0 & 80 \\
2 & 40 \\
4 & 20 \\
6 & 0 \\
\hline
\end{array}
\]
\( (4, 40) \)

The constant of proportionality is \( 160 \).
Solve. Show your work.

15. \( y \) is inversely proportional to \( t \), and \( y = 12 \) when \( t = 7 \).
   a) Find the constant of proportionality.
   \[ k = \frac{84}{7} = 12 \]
   b) Write an inverse equation relating \( s \) and \( t \).
   \[ s = \frac{84}{t} \quad \text{or} \quad st = 84 \]
   c) Find the value of \( s \) when \( t = 5 \).
   \[ s = \frac{84}{5} = 16.8 \]

16. \( y \) is inversely proportional to \( x \), and \( y = 6 \) when \( x = 7.5 \).
   a) Find the constant of proportionality.
   \[ xy = 7.5(6) = 45 \]
   \[ k = 45 \]
   b) Write an inverse equation relating \( x \) and \( y \).
   \[ y = \frac{45}{x} \quad \text{or} \quad xy = 45 \]
   c) Find the value of \( y \) when \( x = 2 \).
   \[ y = \frac{45}{2} = 22.5 \]

17. The density of a substance is the mass of the substance per unit of volume. The density of the element Americium is inversely proportional to its volume. The graph shows the relationship between the density of Americium, \( \rho \) grams per cubic centimeters, and its volume, \( v \) cubic centimeters.
   a) Use the graph to determine the constant of proportionality. Then write an inverse proportion equation.
   \[ \rho = \frac{12}{v} \quad \text{or} \quad \rho v = 12 \]
   b) Explain what the constant of proportionality represents in this situation.
   The mass of Americium
   c) Explain what the point \( (3, 4) \) represents in this situation.
   The density of 3 cm\(^3\) is 4 g/cm\(^3\).
18. A tank has a fixed capacity. The time it takes to fill the empty tank, $t$ minutes, is inversely proportional to the rate of water flowing into the tank, $r$ pints per minute. The graph shows the relationship between $r$ and $t$.

a) Use the graph to determine the constant of proportionality. Then write an inverse proportion equation.

$$ r = \frac{600}{t} \quad \Rightarrow \quad k = 600 $$

b) Explain what the point $(2, 300)$ represents in this situation.

The tank will be filled in 2 min when the rate $= 300$ pt/min.

c) How much time will it take to fill the empty tank if the water is flowing at a rate of 150 pints per minute?

4 min

19. The number of hours, $y$, it takes to drive from Town P to Town Q is inversely proportional to the average speed of a car, $x$ miles per hour. It takes Jeffrey $3 \frac{1}{2}$ hours to drive from Town P to Town Q at an average speed of 60 miles per hour on a particular day. How long will it take Jeffrey to travel from Town P to Town Q if his average driving speed is 70 miles per hour instead?

$$ y = \frac{210}{x} = \frac{210}{70} \Rightarrow 3 \text{ hrs} $$

20. The length of time, $y$ hours, it takes to put a jigsaw puzzle together is inversely proportional to the number of children, $x$, working on it. It takes 16 children 6 hours to put the jigsaw puzzle together. How many children are needed to put the same jigsaw puzzle together in 4 hours?

$$ xy = k \quad \Rightarrow \quad \frac{16 \cdot 6}{64} = \frac{x \cdot 4}{64} $$

24 children