

Systems Solving—High Jump Records

For thirty years, the world records in high jump for both men and women increased consistently, with women's records increasing faster than men's. If the pattern continued, would the records someday be the same?

While systems of two variables can be solved graphically, most systems in the real world contain more than two variables and cannot be solved with a two-dimensional graph. For those problems, there are other solution methods. You will use those methods for two variables and use the graph to check how well they work.

Q1 By substituting, we can use two known facts to conclude a third fact. For example, if Carl owns the book and the person that owns the book wants to sell the book, then what fact can we conclude?

One approach to solving the system of equations is substitution, which is allowed by the transitive property of equality: If $a = b$ and $a = c$, then $b = c$.

Q2 If $y = 3x + 2$ and $y = 5x - 12$, then what does the transitive property of equality allow us to conclude? What is the solution of this equation?

INVESTIGATE

1. Open **High Jump.ftm**. You will see collections and graphs for both men's and women's records for competition high jump since 1970.

Q3 When these data were collected, what was the current record for women, when was it set, and who set it?

To make predictions and comparisons, you need to identify the pattern.

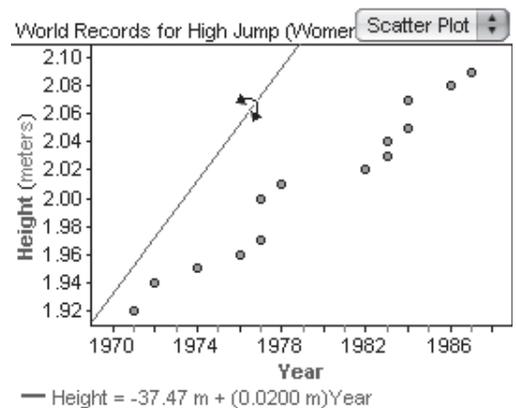
2. Select the graph for the women's records and add a movable line. Position the line to fit the data well.

Q4 What is your equation? What does the slope of this graph tell you? At what rate is the women's record increasing?

3. Repeat this process for the men's records.

Q5 What is your line of fit for the men's records? At what rate is the men's record increasing?

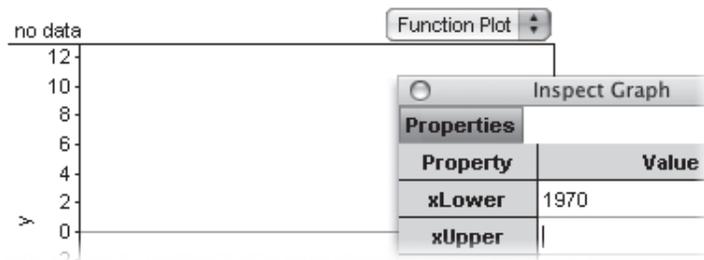
Q6 To find where the lines might intersect, use substitution to equate the expressions in the two height equations. Solve this new equation and give a meaning to your solution.



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continued

- Q7** Substitute your answer to Q6 into each line-of-fit equation you are using from Q4 and Q5. What are the two results, and what do they mean?
4. Add a new graph to the document and choose **Function Plot** from the graph's pop-up menu.
5. Open the graph inspector and set x_{Lower} to 1970 and x_{Upper} to a value more than your answer to Q6. Then set y_{Lower} to 1.9 and y_{Upper} to a value more than your answer to Q7.

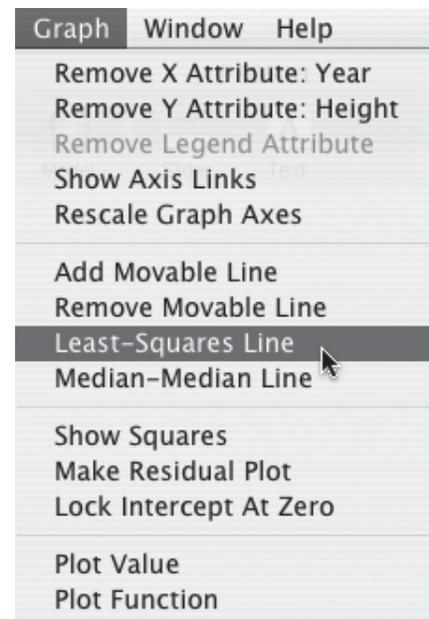


6. Choose **Plot Function** from the **Graph** menu and enter the equation from the women's graph, changing *year* to *x*. Then plot another function using the equation from the men's graph.
- Q8** What do you see in this graph? What's the point of intersection?
- Q9** Does the point you found make sense? Explain.

EXPLORE MORE

You may have noticed that other students found answers quite a bit different from your answer. These two trend lines have similar slopes. Classmates using only slightly different slopes may get quite different answers. By rotating your movable lines the least bit, you can move the intersection by 50 years or more.

One way to avoid variability between predictions is to have everyone use a specific method to find the line of fit. Fathom has two methods for automatically finding lines of fit. Return to the investigation, but this time do not select the movable line. Instead go to the **Graph** menu and choose either **Least-Squares Line** or **Median-Median Line**. Complete the investigation using these equations.



Objective: Students will use substitution to solve a system of linear equations modeling a real-world problem. They will interpret the results and decide on the solution's reasonableness.

Student Audience: Algebra 1

Activity Time: 30–40 minutes

Setting: Paired/Individual Activity (use **High Jump.ftm**)

Mathematics Prerequisites: Students can interpret the slope of a line.

Fathom Prerequisites: Students can add a movable line and create function plots on a graph.

Fathom Skills: In Explore More, students will learn how to add a least-squares line and a median-median line.

Notes: This activity gives students graphic, real-world interpretation of solving a system by substitution. Although the activity explains what substitution is and relates it to other concepts, it does not teach equation-solving steps associated with the substitution method.

As you observe pairs working through the investigation, ask questions that will help students think about what they are doing. You might ask them to explain why the answers for Q7 are the same for both equations and what that means in the context of the problem.

As students share their answers and their reasoning, discuss both the variability in the predictions as well as the unlikelihood that the trend will continue.

Q1 Carl wants to sell the book.

Q2 $3x + 2 = 5x - 12$; $x = 7$

INVESTIGATE

Q3 Stefka Kostadinova set the record in 1987 at 2.09 m.

Q4 Equations will vary. One good equation is $-18.94 + 0.0106 \text{ Year}$. The slope is the average rate of increase (in meters) in the record per year. For women, it is about 0.01 m/yr, or about 1 cm/yr.

Q5 Equations will vary. One good equation is $-14.79 + 0.00866 \text{ Year}$. The rate of increase is 0.008 or 0.009 m/yr (8–9 mm/yr).

Q6 Answers will vary depending on equations: For the equations in Q4 and Q5,
 $-18.94 + 0.0106 \text{ Year} = -14.79 + 0.00866 \text{ Year}$
 The solution to this equation is year 2139, but other answers may differ by 50 years or more. If the trend over the 30 years in these data continues, this is the year that men and women would have nearly the same record high jump.

Q7 The answers should be the same for both equations. The equations above give 3.74 m, but answers within 25 cm are acceptable. This is the record height both men and women would be jumping in the year 2139.

Q8 The graph shows two nearly parallel lines intersecting at about (2139, 3.74). Student answers will vary by 50 years or more.

Q9 This is a highly unlikely prediction. Just because the trends have been fairly linear for 30 years does not mean they will continue in that pattern for another 150 years (nearly five times as long). In fact, records before 1970 show that the present linear pattern was preceded by a definite nonlinear pattern much longer in length.

EXPLORE MORE

Least-squares: $y = -18.184 + 0.010201x$,
 $y = -13.934 + 0.008227x$; (year 2153, record 3.78 m)

Median-median: $y = -21.74 + 0.012x$,
 $y = -17.45 + 0.01x$; (year 2145, record 4.00 m)

