Slope—Runners

At the start of a long race, runners try to set a constant pace that they can maintain throughout the race. In this activity, you'll explore the graphs of data from runners.

Q1 Imagine a graph showing the distance a runner has traveled versus the time he has been running. What sort of pattern would you expect?

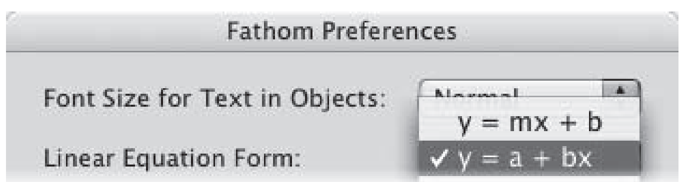
INVESTIGATE

1. Open **Runners.ftm.** You'll see two scatter plots. In the first, a machine has recorded locations (in meters) of three racers each second for 5 s (starting at 0 s).

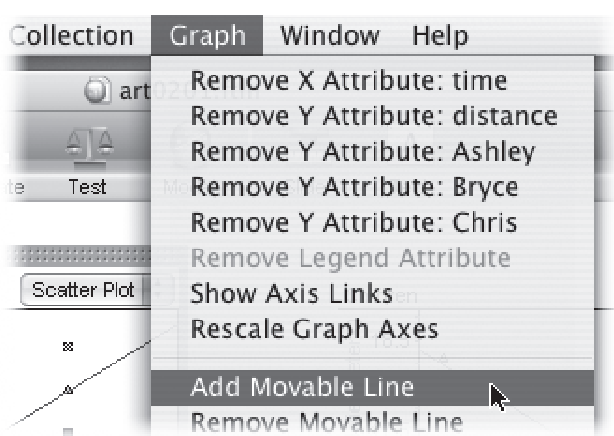
Q2 Move the cursor onto the point that represents Ashley's distance after 1 s. Notice that the coordinates of this point, (1 s, 3.9000 m), appear in the status bar at the lower left corner of the Fathom window. What does this information tell you?

Q3 Move the cursor to the point that represents Ashley's distance after 5 s. Use the coordinates to determine how far Ashley traveled in 5 s.

2. Choose **Preferences** from the **Edit** menu (Windows) or from the **Fathom** menu (Macintosh). Choose $y = a + bx$ for the Linear Equation Form.



3. Click on the first scatter plot. From the **Graph** menu, choose **Add Movable Line.** A line will appear on the scatter plot. In the **Graph** menu, make sure **Lock Intercept At Zero** is turned on.

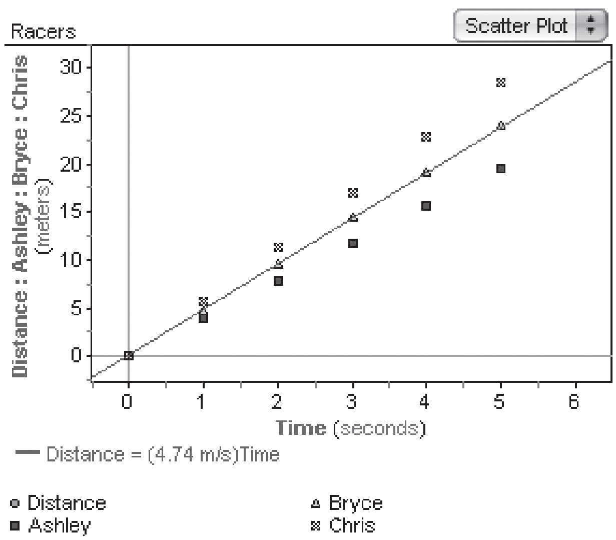


The rotation symbol looks like this.

4. Your first goal is to adjust the movable line until it passes through Ashley's points (the solid squares). When you put your cursor over the line, the cursor will turn into a rotation symbol. Drag the line to rotate it. Below the graph, an equation of the line (*distance* = 0 m + . . .) changes.

Q4 What is the equation when the line passes through the origin and the point representing Ashley's distance at 1 s?

5. Rotate the line to pass through the origin and the point representing Bryce's distance at 1 s.



Q5 What is the equation when the line passes through the origin and the point representing Bryce's distance at 1 s?

Q6 What is the equation when the line passes through the origin and the point representing Chris's distance at 1 s?

Q7 How could you tell from the equations the distances the runners traveled in the first second?

The coefficient of time in the equation is called the *slope* of the line. When the equation represents motion, the slope is the same as the speed (in m/s for this case).

Q8 What does the slope indicate about the graph of the line?

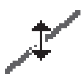
Q9 How could you find the slope of one of these lines from the coordinates of the data point it goes through at time 2 s? At time 4 s?

Q10 What would the line look like for someone who ran faster than Chris? Slower than Ashley? What would the coefficient of *time* look like for someone who ran faster than Chris? Slower than Ashley?

After the race, the machine started accidentally and made two records of the locations of two children running in different directions along the track. This is the second scatter plot in the Fathom document.

6. Select this graph and add a movable line. Because the origin isn't showing on this graph, you won't lock the intercept at zero.

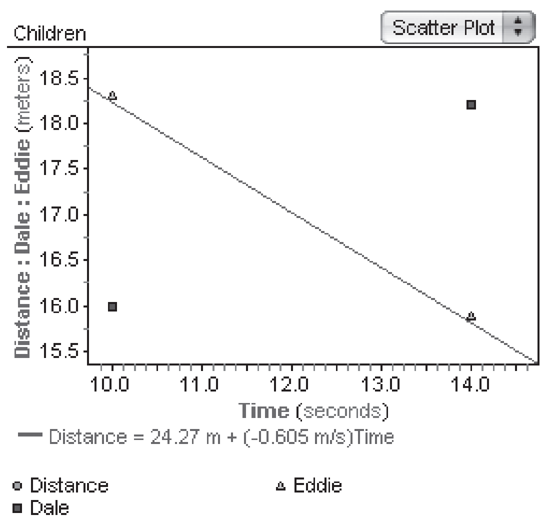
When you put your cursor over the middle of the line, it turns into an up-and-down arrow. If you click on that part of the line, you can drag the line up and down.



7. Adjust the movable line to pass through Dale's data points (the squares). You can rotate the line as before, or because it is not locked at zero, you can drag it up and down. The equation for the line changes as you move the line.

Q11 Dale's speed shows up in the equation as the slope of the line. What was Dale's speed?

8. Adjust the movable line to pass through Eddie's data points. (You'll need to do some extreme rotating.)



Q12 What was Eddie's speed? What does it mean that the slope of this line is negative?

Q13 In general, how can you find the slope of a line through two given points?

Q14 An equation of Eddie's line is approximately

$$\textit{distance} = 24.3 + (-0.604)\ \textit{time}$$

What does the first number (the one that's not a coefficient) represent in all the equations you've considered?

Objective: Students will come to understand slope as a constant speed and as the steepness of a line. Students will learn that a line with a negative slope drops from left to right, and they will calculate the slope of a line by dividing the vertical change by the horizontal change (from zero).

Student Audience: Pre-algebra, Algebra 1

Activity Time: 25–35 minutes

Setting: Paired/Individual Activity or Whole-Class Presentation (use **Runners.ftm** for either setting)

Mathematics Prerequisites: Students can read the coefficient of a variable from an equation and understand coordinates of points in the rectangular coordinate system.

Fathom Prerequisites: Students can start Fathom and open a document.

Fathom Skills: Students will learn how to set the linear equation form, add a movable line, lock the line's intercept at zero, adjust the line, and read the equation.

Notes: Many students are attracted to the racing idea and may want to discuss the reasonableness of the speeds involved. There may also be questions about how constant the speeds can be if time 0 means when the runners begin. You can use the activity to bring out that slope will always be some kind of rate (though not always a speed) and will always have units such as meters per second, miles per gallon, or degrees Fahrenheit per foot below the surface.

As you circulate, ask students to describe how the equation for the line changes as they rotate the line. Ask further "what if" questions, such as those in Q10. Give students a chance to share with the whole class their answers and the reasoning behind their answers. Select different pairs to share their answers and thinking for Q7–Q13.

For a Presentation: You might use this activity to introduce Fathom and some of its features. To deepen students' understanding of the relationship between the equation and the line representing that equation, ask several students for answers to Q7–Q10 and Q13.

Q1 The points should lie in a line, because if the speed is constant, then time and distance should be proportional.

INVESTIGATE

Q2 Ashley ran 3.9 m the first second.

Q3 Ashley ran 19.5 m in 5 s.

Q4 Answers should be approximately *distance* = (3.9 m/s)*time.*

Q5 Answers should be approximately *distance* = (4.8 m/s)*time.*

Q6 Answers will vary. They should be approximately *distance* = (5.7 m/s)*time.*

Q7 The distances after 1 s are the coefficients of *time* in the equations.

You might reiterate that this number is called the *slope* and add that it's also the *rate.* The term *slope* applies to the graph because it describes the steepness. The term *rate* applies to the problem situation. In this case, it is a rate of meters per second, which is a speed.

Q8 The slope represents the steepness of the line.

Q9 Divide the distance coordinate (the change in distance from 0) by 2, which is the change in time during the first two seconds. Divide the distance coordinate by 4.

Q10 The line for a faster runner would be steeper; for a slower runner, the line would be less steep, or flatter. The coefficient (slope) would be larger for a faster runner and smaller for a slower runner.

Q11 Answers will be close to Dale's actual speed of 0.55 m/s.

Q12 Answers will be close to Eddie's actual speed of −0.6 m/s. Eddie moved in the direction opposite to the other runner's direction. The line is dropping from left to right.

Q13 Explanations should be equivalent to dividing the change in vertical coordinates by the change in horizontal coordinates. Students may say that they're dividing the vertical coordinate of a point not on the vertical axis by the horizontal coordinate of that point, because they may only be considering the case where one point is (0, 0).

Q14 The constant term is the *y*-intercept—the point where the line crosses the vertical axis. In these examples, it is the position of the runner when the time was 0. It can be thought of as the height from which a point rises or falls as it moves from 0 to the right along the line.

EXTENSION

Students can make their own measurements, enter them into a Fathom case table, and look for lines of fit that will allow them to make predictions. They might measure motion, change in temperature, or other rates.