WARM UP: Provide a closed form for the following indefinite sum.

\[
\sum_{k=0}^{n} \left( \frac{1}{5} \right)^k
\]

\[
\sum_{k=1}^{n} \left( \frac{1}{5} \right)^k
\]

\[
S = \frac{1}{5} \cdot \sum_{k=0}^{n} \left( \frac{1}{5} \right)^k
= \frac{1}{5} \cdot \frac{1 - \left( \frac{1}{5} \right)^{n+1}}{1 - \frac{1}{5}}
= \frac{5}{4} \left( 1 - \frac{1}{5^{n+1}} \right) - 1
\]

Suppose you have a mystery function \( f \) and all you know is a formula for its associated series.

\[
\sum_{k=0}^{n} f(k) = n^2 - 3n
\]

5. Find each sum.

\[\begin{align*}
\text{a. } & \sum_{k=0}^{10} f(k) \\
\text{b. } & \sum_{k=5}^{10} f(k) \\
\text{c. } & \sum_{k=1}^{11} f(k - 1) \\
\text{d. } & \sum_{k=0}^{10} (3f(k) + 5^k)
\end{align*}\]
12. Evaluate each definite sum.

a. \[ \sum_{j=12}^{45} (4j + 3) \]

b. \[ 5 + 9 + 13 + 17 + 21 + \cdots + 201 \]

c. \[ \sum_{k=1}^{36} k(k + 1) \]

d. \[ 1 - 2 + 3 - 4 + 5 - \cdots - 1000 \]

e. \[ 1 - 2 + 4 - 8 + 16 - \cdots + 1024 \]

\[
\sum_{k=1}^{36}(k^2 + k) = \left( \sum_{k=1}^{36} k^2 \right) + \left( \sum_{k=1}^{36} k \right)
\]

\[
\frac{36(37)(73)}{6} + \frac{36^2 + 36}{2} = 16200 + 1260 = 17460
\]

a. \[ \sum_{j=12}^{45} (4j + 3) \]

b. \[ 5 + 9 + 13 + 17 + 21 + \cdots + 201 \]

c. \[ \sum_{k=1}^{36} k(k + 1) \]

d. \[ 1 - 2 + 3 - 4 + 5 - \cdots - 1000 = -500 \]

e. \[ 1 - 2 + 4 - 8 + 16 - \cdots + 1024 \]

\[ S = 1 + 2048 \]

\[ S = \frac{1 + 2048}{3} \]
13. Below is an illustration of the first three pyramidal numbers. Each layer is a square.

a. Use this illustration to list the first five pyramidal numbers.
b. What series does the total number of dots in each figure represent? Is this series an arithmetic series? Explain.
c. Find the closed form for the number of dots in a pyramid that is \( n \) rows high.
d. How many dots are in a pyramid ten levels high?

\[
\begin{align*}
th_{10} &= \sum_{k=1}^{10} k^2 = \frac{10(11)(21)}{6} \\
\sum_{k=1}^{n} k^2 &= \frac{n(n+1)(2n+1)}{6}
\end{align*}
\]

d. \( g(1) = 5, \ r = \frac{1}{2} \)
e. \( g(5) = 18, \ g(6) = 9 \)
f. \( g(3) = 12, \ g(5) = 30 \)
g. \( g(6) = 6, \ g(8) = 54 \)
h. \( g(5) = 6, \ g(9) = 54 \)

\[
\begin{align*}
g(n) &= 18 \left(\frac{1}{2}\right)^{n-5} \\
g(n) &= 576 \left(\frac{1}{2}\right)^{n-5} \\
18 &= a \left(\frac{1}{2}\right)^5
\end{align*}
\]
Mary loves cake. In order to make the cake last a long time she eats 1/3 of the cake the first day and 1/3 of what was left each day after.

a. How much cake did she eat on the 10th day?

b. How much cake has she eaten after 10 days?

c. n days?
Her brother Harry has a birthday a few days after her and he too likes cake. But he eats $\frac{2}{3}$ of the cake the first day and $\frac{2}{3}$ of what is left cake each day after that. How do the answers change?

\[
\sum_{k=1}^{n} \frac{2}{3^k} = 1
\]
Minds in Action episode 36

Tony, Sasha, and Derman are working on the following questions.

**Question 1** Suppose g is a geometric sequence with first term \( g(0) = 1 \) and common ratio \( \frac{1}{2} \). Does \( g \) have a limit? If so, what is it?

**Question 2** Using the same function \( g \), does the associated series have a limit? If so, what is it?

**Sasha** Let’s try making a table for \( g \) so we can see what’s going on.

**Tony** I’ve already filled in values up to \( n = 3 \).

**Derman** Look—the outputs get smaller and smaller! They seem to approach 0, so I guess that’s the limit.

**Sasha** It’s not enough to say “they seem to approach 0.” In order to say that the limit of the sequence is 0, we have to show that we can get as close as we want to 0. I mean, if we pick any number close to 0, we then have to find an \( n \) such that \( g(n) \) is even closer to 0 than our number.
Tony

Yes, look at this.

To get within 0.1 of 0, let \( n = 2 \).

\[ g(2) = \left( \frac{1}{3} \right)^2 = \frac{1}{9} < 0.1 \]

To get within 0.01 of 0, let \( n = 3 \).

\[ g(3) = \left( \frac{1}{3} \right)^3 = \frac{1}{27} < 0.01 \]

To get within 0.001 of 0, let \( n = 5 \).

\[ g(5) = \left( \frac{1}{3} \right)^5 = \frac{1}{243} < 0.001 \]

Since you can get as close as you want to 0 by making \( n \) large enough, 0 is the limit of the sequence.

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**Chapter 7  Sequences and Series**

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**For You to Do**

Suppose \( a = 12 \). Pick three values for \( r \) from this list.

\[ 2, \frac{1}{2}, \frac{1}{3}, -1, -\frac{1}{2}, -\frac{1}{3}, 3, 9, 8, 10 \]

6. For each \( r \)-value you select, decide whether the following series has a limit.

\[ \sum_{k=0}^{n} g(k) = \frac{12}{1 - r} \left( 1 - r^{n+1} \right) \]

7. If it does, say what the limit is. If it does not, explain why not.

8. If \( r = 1 \), what happens with the formula? Does the series have a limit? Explain.

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**For Discussion**

9. Explain how you can tell, just based on the value of \( r \), if a geometric series has a limit.

10. Can an arithmetic series have a limit? Does the limit depend on the first term, the common difference, or both? Explain.
Do you remember this exercise from Lesson 7.1?
A wealthy relative left Emile an inheritance. The will read as follows.
To Emile, because he likes math problems, I leave a choice. He can have one of two inheritances.

**Option 1** A starting amount of $10,000 with an annual payment at the end of each year that is $1000 more than the amount he received the previous year, for the next 26 years

**Option 2** A starting amount of one cent with an annual payment at the end of each year that is twice the amount he received in the previous year, for the next 26 years

To find the total Emile gets under Option 1, you can sum an arithmetic series. After 26 years, he will get

\[ 10,000 + 11,000 + 12,000 + 13,000 + \cdots + (10,000 + 1,000n) \]

To find the total Emile gets under Option 2, you can sum a geometric series. After 26 years, he will get

\[ 0.01 + 0.02 + 0.04 + 0.08 + \cdots + (0.01) \cdot 2^n \]

15. a. What is \( n \)? That is, what is the last term you would add in each series?
   b. Sum the arithmetic series to find the total Emile gets with Option 1.
   c. Sum the geometric series to find the total Emile gets with Option 2.

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**7. Standardized Test Prep** What is the limit of the following geometric series?

\[ 1 + \frac{3}{5} + \frac{9}{25} + \frac{27}{125} + \frac{81}{625} + \cdots \]

A. \( \frac{3}{2} \)  
B. 2  
C. \( \frac{5}{2} \)  
D. 3

\[ S = 1 + \frac{3}{5} + \frac{9}{25} + \cdots + \left(\frac{3}{5}\right)^n \]

\[ \frac{3S}{5} = \frac{3}{5} + \frac{9}{25} + \cdots + \left(\frac{3}{5}\right)^{n-1} \]

\[ \frac{3S}{5} = 1 - \left(\frac{3}{5}\right)^{n+1} \]

\[ |r| = 1 \text{ no limit} \]

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Nov 30-6:58 AM

Nov 30-7:00 AM